A note of controlled Rolling In-Hand Manipulation on Geodesic Curves

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Abstract

In hand manipulation of object by robotic hands has been in focus since decades [1]. In-hand manipulation by robotic fingers involve rolling motion [2, 3], sliding motion, sticking motion etc. Rolling motion of the fingertip on the object surface is one of the methods to manipulate the object. Different methodologies have been developed to control the object motion by rolling of fingertips. The motion of the fingertips control both the gross motion of the object and the fine motion of the fingertip on the object surface. The fine motion results in the generation of contact curves on both the object and the fingertips. However, for a rolling manipulation, the development of contact curves on the object and fingertip is related by rolling constraints. This paper focuses on presenting the development of a reduced order dynamics equation of motion during in-hand manipulation. We prove that for rolling constrained manipulation, if the contact curves are limited to geodesic curves on the object surface, then the equation of motion is represented as a first order differential equation in terms of the contact curve coordinates. It is also shown that the geodesic based contact curves on the surface of the object results in the geodesic based contact curves on the surface of the fingertip as well. Hence, the constraint forces only depend on the first order derivatives of the contact coordinates, and hence the instantaneous twist vector of the object as well as the fingertip. An example is presented via simulation where the gross position of the object and the fine contact curves on the object surface are controlled on a sphere by two robotic fingertips. Here, the important steps are shown to ensure brevity.

Consider an object being manipulated by robotic rolling contacts. The equation of the surface is given by $g_o(u_o, v_o) = 0$. The equation of the surface of the fingertip is given by $g_{f_i}(u_{f_i}, v_{f_i}) = 0$. The terms u, v represent the local coordinates on the object surface. Let the modified form of the Montana's constraint equation [4] is given by:

$$\mathbf{A}\dot{\mathbf{q}} = \dot{\mathbf{U}} \tag{1}$$

where $\dot{\mathbf{q}} = [\dot{\mathbf{q}}_{\mathbf{f_1}}, ..., \dot{\mathbf{q}}_{\mathbf{f_k}}, \dot{\mathbf{q}}_{\mathbf{o}}]$, $\mathbf{q_{fi}}$ represents the twist vector for the i^{th} finger and $\mathbf{q_o}$ represents the twist vector for the object. $\mathbf{A}$ matrix contain the surface property terms. The non-holonomic rolling constraint is given by $\mathbf{B}\dot{\mathbf{U}} = 0$ and the contact-constraint of object and the fingertips is represented by $\mathbf{C}\dot{\mathbf{q}} = 0$. The $\mathbf{B}$ matrix is composed of the terms relating the second order properties of the surface of the object and the finger-tip. The combined constraint equation for the rolling motion is represented as: $\mathbf{D}\dot{\mathbf{q}} = 0$, where $\mathbf{D} = [\mathbf{C}\ \mathbf{AB}]^T$. The net equation of motion is given by:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{V}(\dot{\mathbf{q}}, \mathbf{q}) + \mathbf{g}(\mathbf{q}) = \mathbf{u_i} - \mathbf{D^T}\lambda \tag{2}$$

where $\mathbf{M}$ is the generalised mass matrix, $\mathbf{V}(\dot{\mathbf{q}}, \mathbf{q})$ contains the coriolis terms, the vector $\mathbf{g}(\mathbf{q})$ represents the gravity terms and the vector $\mathbf{u_i}$ is the set of external wrenches on the fingertip and the object. On the object with contact coordinates as u_o, v_o, the geodesic equation [5] on the object surface is given by:

$$\begin{aligned}
\frac{d^2u_o}{ds^2} + \Gamma^1_{11}\frac{du_o}{ds}\frac{du_o}{ds} + 2\Gamma^1_{12}\frac{du_o}{ds}\frac{dv_o}{ds} + \Gamma^1_{22}\frac{dv_o}{ds}\frac{dv_o}{ds} = 0 \\
\frac{d^2v_o}{ds^2} + \Gamma^2_{11}\frac{du_o}{ds}\frac{du_o}{ds} + 2\Gamma^2_{12}\frac{du_o}{ds}\frac{dv_o}{ds} + \Gamma^2_{22}\frac{dv_o}{ds}\frac{dv_o}{ds} = 0
\end{aligned} \tag{3}$$

where Γ^k_{ij} is the second-order christoffel symbol. The variation of path parameter is assumed as $\frac{ds}{dt} = \sigma$. On a surface, the time parameterised geodesic equation is represented as:

$$\ddot{\mathbf{U}}_\mathbf{o} + \mathbf{E_o}\mathbf{K_o} = \frac{\sigma}{\dot{\sigma}}\left(\mathbf{J_o}\dot{\mathbf{U}}_\mathbf{o} - \mathbf{J_i}\dot{\mathbf{U}}_\mathbf{i}\right) \tag{4}$$

where $\mathbf{U}_i = [u_i, v_i, u_{o_i}, v_{o_i}]$, and u_i, v_i represents the local surface coordinates on the i^{th} finger surface and the corresponding coordinates at the i^{th} contact point on the object. In this paper, we prove that if the fingertip rolls on the surface of the object and the contact curve on the object surface is given by equation 4, then the contact curve on the surface of the fingertip is given by equation 5.

$$\ddot{\mathbf{U}}_\mathbf{i} + \mathbf{E}_\mathbf{i}\mathbf{K}_\mathbf{i} = \frac{\sigma}{\dot{\sigma}}\left(\mathbf{J}_\mathbf{i}\dot{\mathbf{U}}_\mathbf{i} - \mathbf{J}_\mathbf{o}\dot{\mathbf{U}}_\mathbf{o}\right) \quad (5)$$

The matrix $\mathbf{E_i}, \mathbf{E_o}$ contain the terms relating the local surface properties of the object and the fingertip. The vector $\mathbf{K_i}$ contains the squared terms for the i^{th} fingertip $(\dot{u}_i^2, \dot{u}_i\dot{v}_i, \dot{v}_i^2)$ and the vector $\mathbf{K_o}$ contains the squared terms for the surface coordinates of the object $(\dot{u}_o^2, \dot{u}_o\dot{v}_o, \dot{v}_o^2)$. Combining the equation 1-5, the reduced dynamics equation of motion in terms of the object surface coordinates is given by:

$$\mathbf{MD}^+(\mathbf{K}(\mathbf{U},\dot{\mathbf{U}}) - \dot{\mathbf{D}}\mathbf{D}^+\dot{\mathbf{U}}) + \mathbf{V}(\dot{\mathbf{q}},\mathbf{q}) + \mathbf{g}(\mathbf{q}) = \mathbf{u_i} - \mathbf{D}\lambda \quad (6)$$

where, $\mathbf{K}(\mathbf{U},\dot{\mathbf{U}})$ is a matrix derived by utilising equation 4 in the dynamics equation 2. The constraint forces λ is given by:

$$\lambda = \mathbf{D^T}^+(\mathbf{MD}^+(\mathbf{K}(\mathbf{U},\dot{\mathbf{U}}) - \dot{\mathbf{D}}\mathbf{D}^+\dot{\mathbf{U}}) + \mathbf{V}(\dot{\mathbf{q}},\mathbf{q}) + \mathbf{g}(\mathbf{q}) - \mathbf{u_i}) \quad (7)$$

The constraint forces only depend on the variation of $\mathbf{U}$ and $\dot{\mathbf{U}}$. This is possible only if the finger constrains geodesic based contact curve on the object surface as the geodesic based contact curves relate the second order growth of contact coordinates to the first order growth. Only the contact coordinates on the surface of the object needs to be controlled. The rolling constraint relates the contact curve on the object to the contact curve on the fingertip.
The geodesic curve only relate the instantaneous acceleration to the instantaneous velocity of the object. Hence, it does not constrain the set of twists available to the object. At a given grasp configuration, the set of available twists is given by the null space of the $\mathbf{D}$ matrix. In order to control the gross motion of the object and the fine contact curves on the object surface to be geodesic, the feedback linearization method (presented in [3]) was used. As an example, two fingers manipulating an object is simulated using feedback linearization method. The contact curves on the object surface are geodesic curves and the motion of the object is controlled. The motion of the object is shown in Figure 1(a). The variation of the constraint forces is presented in Figure 1(b).

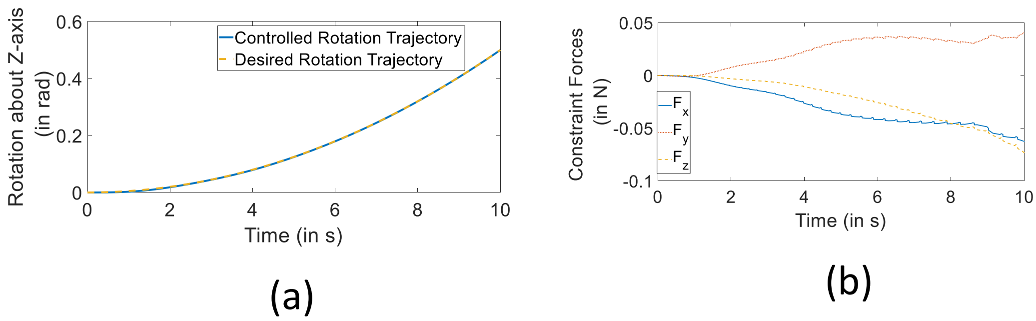

Figure 1: (a) The desired rotation of the object and the controlled trajectory in the z-axis (b) Corresponding reaction forces

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