

## Dynamics of Single-Link Flexible Manipulator: Theoretical and Experimental Study

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### Abstract

Due to requirements of lightweight, larger work volume, higher payload to manipulator weight ratio, lower energy consumption, higher operational speed and to improve productivity in an industry, the flexible manipulators are becoming a viable choice. Hence, It is crucial to accurately simulate the flexible systems to understand their behavior. Various methods that were used to formulate and simulate flexible multibody systems have been reviewed in the papers [1,2,3] extensively. It is clear after the review that there is a need for further improvement in terms of numerical stability, computational complexity, efficiency etc. This work presents the dynamic modeling of a Single-Link Flexible Manipulator (SLFM) using the Decoupled Natural Orthogonal Complement (DeNOC) matrices [4] and its experimental validation. The flexible link, assumed as the Euler-Bernoulli beam, is discretized using the assumed mode method to represent the link deflection. Further, the transverse displacement with 2-modes are considered for the flexible link neglecting the axial deflection and torsional effect. The  $i^{\text{th}}$  flexible link of length  $a_i$  for the current problem is shown in Fig. 1(a), where  $\bar{u}_i$  is transverse deflection and the experimental setup is shown in the Fig. 1(b).

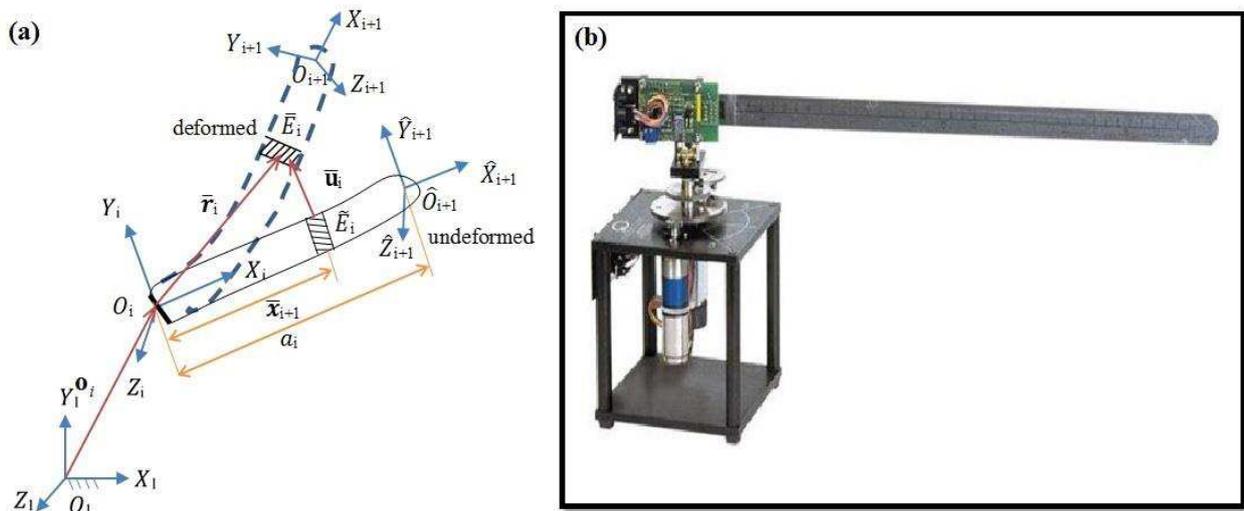


Figure 1: (a) Flexible link for dynamic modeling (b) Experimental setup [5]

The DeNOC based computationally efficient formulation, which gives the recursive analytical expressions for the matrices and vectors associated with the dynamic equations of motion including the geometric stiffness, is used for rigid-flexible multibody systems. The DeNOC matrices for the flexible link were derived by writing the expressions of linear and angular velocities of the two links as well as the rates of the flexible coordinate of the second link. The parameters for modeling are taken from the experimental setup, which are shown in table 1. The dynamic behavior of the system can be evaluated by (1), where  $\mathbf{I}$  is generalized inertia matrix,  $\ddot{\mathbf{q}}$  is double derivative of generalized coordinate vector,  $\mathbf{h}$  is a vector of convective inertia and  $\boldsymbol{\tau}$  is generalized force vector. The detailed derivation and final expression of these vectors and matrices are given in the full paper.

$$\mathbf{I}\ddot{\mathbf{q}} + \mathbf{h} = \boldsymbol{\tau} \quad (1)$$

The simulations are carried out for two cases, one for the freefall simulation to verify the trend and other for the forced simulation, in which the voltage-torque relation is given input for the experimental and theoretical model. The obtained results from dynamic modeling and experimental setup demonstrate a close agreement for variation in joint angle and lateral deformation of the tip with respect to time.

Table 1: Parameters for flexible link taken from the setup

S.No.	Parameter	Value
1	Length of link ( $a$ )	0.419m
2	Width	$20.73 \times 10^{-3}$ m
3	Thickness	$0.79 \times 10^{-3}$ m
4	Mass per unit length ( $\rho$ )	0.155 kg/m
5	Young's modulus	$170 \times 10^9$ N/m <sup>2</sup>

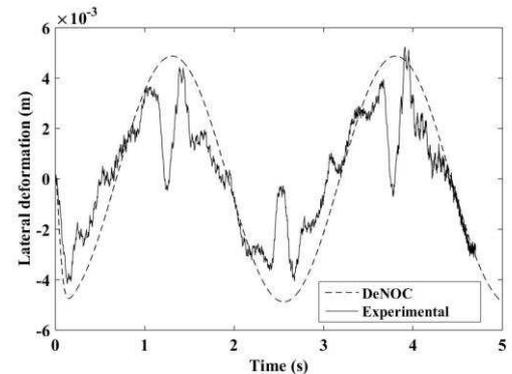


Figure 2: Lateral deformation v/s time

This work provides the proper framework for formulating the dynamic equations of motion and validating it with an experimental setup especially for the beginners or the undergraduates who wanted to analyze flexible systems such as helicopter blades, satellite antennas, etc.

## References

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