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Determination of Point of Contact for Cone-Line and Cone-Cylinder Primitives using Denavit-Hartenberg (DH) Parameters

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Abstract

Conical surfaces are used in tread of wheels of railway vehicles that come in contact with rails. For simplified mathematical modeling of the wheel-rail interaction, a wheelset is considered as a double cone or also referred as dicone [6]. The conical shape ensures that the wheelset has differential effect on a turning and also it helps in re-centering of the wheelset if there is any lateral movement. For simplified modeling, the shape of the rails can be considered as knife-edge (represented as lines) or as cylinders. Similarly rail-guided vehicles such as roller-coaster generally have conical shaped wheels and cylindrical rails and hence the cone-line and cone-cylinder contact point determination is important.

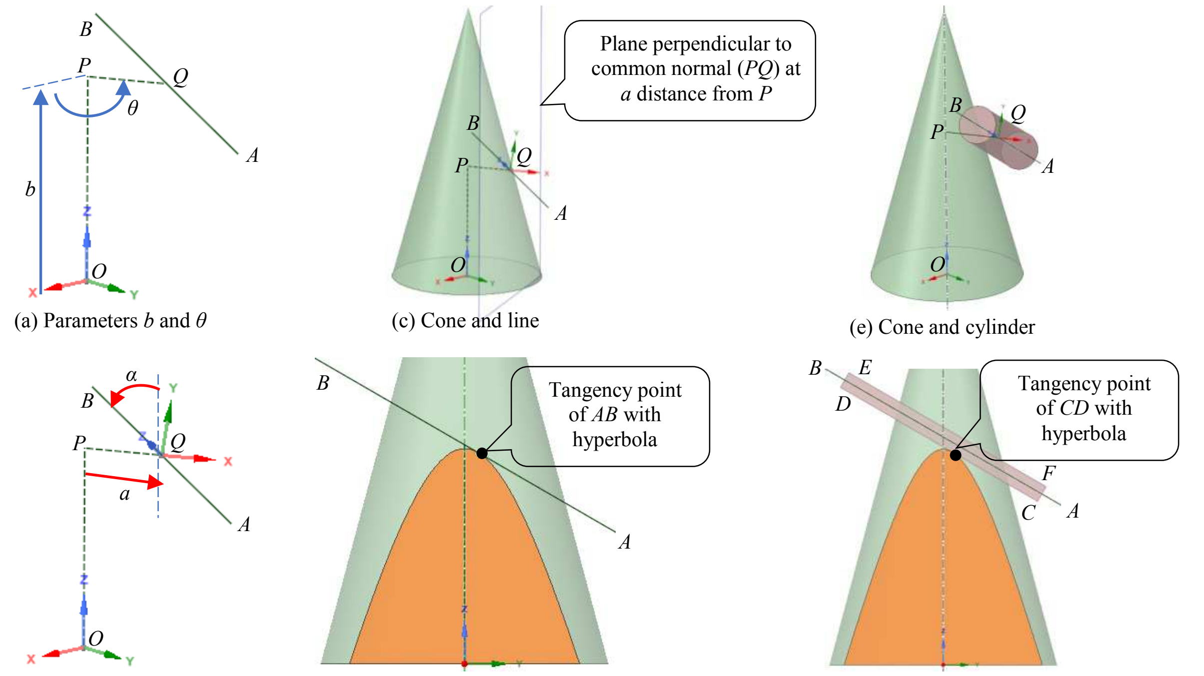
For the contact point determination of a dicone and knife-edge rails, represented using lines, assumptions are made that the contact point lies in the vertical plane, to simplify the calculations [6]. The inaccurate contact point in turn results in inaccurate forces applied at the point and hence the subsequent mathematical model would have incorrect formulations. Hence, determination of exact contact point is important. The contact point is considered not to be in the vertical plane of the wheelset in [2], but the point is still approximated to a first order term, ignoring the higher order terms. Similarly, for a dicone on cylindrical rails, approximated solutions were proposed by De Pater [5]. An exact contact point detection methodology was proposed by Antali et al. [1], which uses parametric surfaces for cone and cylinders and solves for coincidence and tangency conditions at the point of contact. They further derive the exact kinematic motion equations for hunting behavior.

In this paper, novel analytical methods to determine contact or tangency points between a cone and line, and a cone and cylinder, are proposed. A line in 3-dimensional space can be represented with respect to a coordinate frame in multiple ways. Denavit and Hartenberg proposed that four parameters are sufficient to achieve the same [4]. Referring to Figure 1(a), a line AB has to be represented in the frame XYZ. Line that is perpendicular to both Z axis and the line AB is determined and referred to as the common normal. Its intersection with Z axis is denoted with P and with AB as Q. Thereafter, the frame is translated by parameter b along current Z axis to reach P. Then, the frame is rotated about current Z axis to align the X axis along the common normal PQ. The angle rotated is termed as parameter θ. Thereafter, the frame is translated by parameter a to reach the point Q on line AB. Finally, the frame is rotated by parameter α about current X axis (also the common normal) so that its Z axis gets aligned along the line AB. Thus, by using four elementary transformations, one can reach a line from a coordinate frame (Figure 1(b)). These four parameters are referred to as the Denavit-Hartenberg (DH) parameters, which are generally used in robotics to represent a serial robot's architecture. The methods proposed here to determine cone-line and cone-cylinder contact points are similar to the analytical method proposed by the authors to determine collision between cylinders, using DH parameters and dual number algebra [3], where the axis symmetric properties of cylinders were exploited to convert a complicated 3-dimensional problem to simpler planar tests.

The authors propose that a coordinate frame is attached at the base of a cone, with its Z axis aligned with the axis of the cone, as shown in Figure 1(c). Line AB can be represented using the four DH parameters as explained earlier. However, if it has to be touching the conical surface, one of the four DH parameters has to be dependent on the other three parameters. Thus, one constraint equation exists, to ensure contact. To obtain the constraint equation and thereafter the exact contact point, the authors propose a plane perpendicular to line PQ (common normal) and translated by parameter a from P along PQ. Since this

is a plane cutting the cone vertically, the cross-section of the cone is a hyperbola, which can be expressed in terms of parameter a. This is illustrated in Figure 1(d). For contact of the line and the cone, line PQ (in the cross-section plane) has to intersect the hyperbola at one point, being the point of tangency. After solving the geometric and algebraic equations, the authors have obtained the constraint equation and the exact point of contact. Note that the sequence of parameters b and θ can be swapped without altering the further transformations. All the equations were derived with respect to the frame obtained after θ rotation about Z axis. Hence, a complicated 3-dimensional problem was simplified and solved in the cross-section plane, as viewed from the common normal direction, shown in Figure 1(d).

Further, the authors propose a cylinder's axis as line AB and similar approach has to be followed to determine the DH parameters, as illustrated in Figure 1(e). Again, one of the parameters is dependent and hence a constraint equation exists. A plane perpendicular to the common normal at a particular distance u ($\neq a$) from P along PQ cuts the cone as a hyperbola and the cylinder as rectangle $CDEF$, as illustrated in Figure 1(f). The constraint equation has been derived by considering one of the edges (CD) of the rectangle to be tangent to the hyperbola. Thereafter, the exact point of contact has been determined with respect to the frame attached to the cone's base. The proposed methods can further be used to determine the exact point of contacts in dicone and knife-edge or cylindrical rails.



(a) Parameters b and θ (c) Cone and line (e) Cone and cylinder

(b) Parameters a and α (d) Cone cut as hyperbola and its intersection with AB resulting in the point of cone-line contact (f) Cone cut as hyperbola and cylinder as rectangle. Point of contact is intersection of hyperbola and CD

Figure 1: Denavit-Hartenberg (DH) parameters used to represent line with respect to a frame, and to determine point of contact between cone and line, and cone and cylinder

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