Multibody model of railway wheelset using Natural Orthogonal Compliments

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Abstract

In this work, a railway wheelset is modeled as a serial chain system, using the method of Decoupled Natural Orthogonal Compliments (DeNOC). While several methods have been used to model the dynamics of a wheelset [2–5], in DeNOC formulation recursive relations were used to systematically derive the kinematic constraints. Consequently, highly efficient algorithms can be developed for forward and inverse dynamics [1]. Due to curved profile of the wheels special shape constraints are introduced, using which instantaneous roll center was identified. Using the DeNOC method, a two Degree Of Freedom (DOF) wheelset model was developed to study hunting motion.

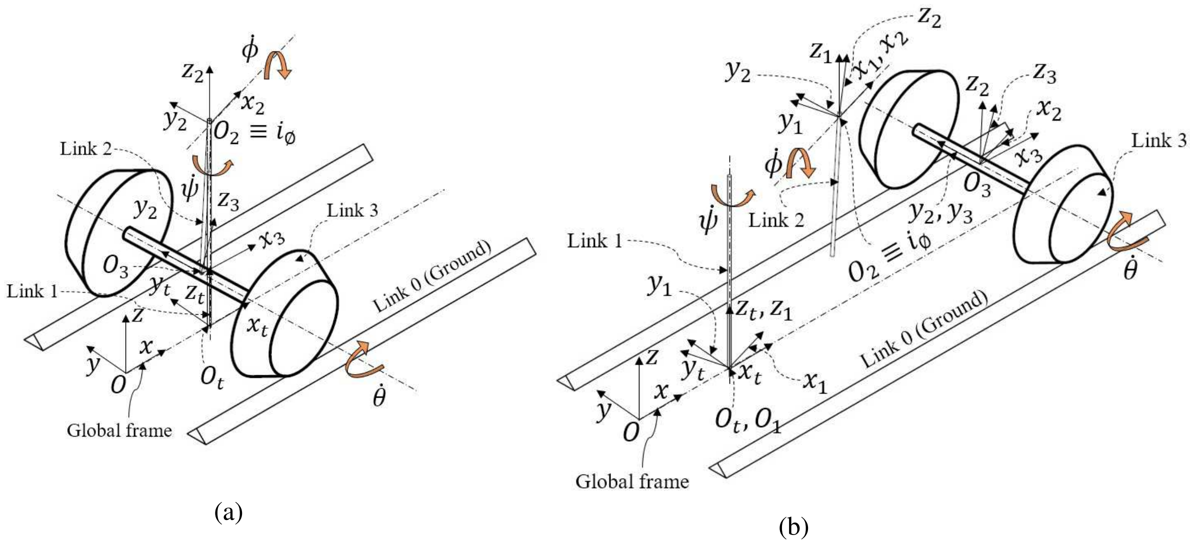


Figure 1: (a) Serial chain wheelset model, (b) Exploded view of serial chain model

A system of four links were used to represent the kinematic constraints of a general wheelset motion, as shown in Figure 1. First link (Link 0) is ground. Origin on the second link (Link 1) 'O_1' is at the instantaneous trajectory coordinate and Link 1 can rotate about 'z_1' axis. It is important to note that the length of link 1 is a time varying quantity since it is dependent on the instant rolling radius of wheels, however for a wheelset with conical profile the link length remains constant. Origin of the third link (Link 2) 'O_2'is situated at the instantaneous roll center 'i_ϕ' and this link can rotate about 'x_2' axis. Finally, link 3 is the wheelset itself, which rotates about 'y_3' axis. Ground and Wheelset are real whereas the intermediate links (Link 1 and Link 2) are imaginary. Compact form of Newton Euler equation of an 'i'th link in a serial chain system can be written as follows

$$M_i\dot{\boldsymbol{t}}_i + W_iM_i\boldsymbol{t_i} = \boldsymbol{w_i} \quad : i = 0, 1...3 \tag{1}$$

Where, $\boldsymbol{t_i}$ and $\boldsymbol{w_i}$ are the twist and wrench vectors of 'i' th body respectively. The twist vector contains angular velocity ($\boldsymbol{\omega_i}$) and linear velocity ($\dot{\boldsymbol{c}}_i$) of center of mass (C_i) and can be expressed as $\boldsymbol{t_i} = H_i\dot{\boldsymbol{q}}_i$, whereas the wrench vector consists of resultant moments ($\boldsymbol{n_i}$) and forces ($\boldsymbol{f_i}$) acting at the center of mass. M_i is the mass matrix. Expression for $\boldsymbol{t_i}$, $\boldsymbol{w_i}$, M_i and W_i are given below

$$\boldsymbol{t_i} = \begin{bmatrix} \boldsymbol{\omega_i} \\ \dot{\boldsymbol{c}}_i \end{bmatrix} = \begin{bmatrix} G_i & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{1} \end{bmatrix} \begin{bmatrix} \dot{\boldsymbol{\gamma}} \\ \dot{\boldsymbol{c}}_i \end{bmatrix}, \boldsymbol{w_i} = \begin{bmatrix} \boldsymbol{n_i} \\ \boldsymbol{f_i} \end{bmatrix}, M_i = \begin{bmatrix} I_i & \boldsymbol{O} \\ \boldsymbol{O} & m_i.\boldsymbol{1} \end{bmatrix}, W_i = \begin{bmatrix} \tilde{\omega}_i & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{O} \end{bmatrix} \tag{2}$$

In equation (2), $\dot{\boldsymbol{\gamma}}$ consists of time derivatives of Euler angles, m_i is the mass, I_i is the inertia matrix and $\tilde{\omega}_i$ is the skew symmetric form of angular velocity vector. $\boldsymbol{O}$ and $\boldsymbol{1}$ are the 3×3 zero and identity matrices respectively.

The unrestrained equation of motion of a serial chain wheelset system is written as

$$M\dot{\boldsymbol{t}} + WM\boldsymbol{t} = \boldsymbol{w} \tag{3}$$

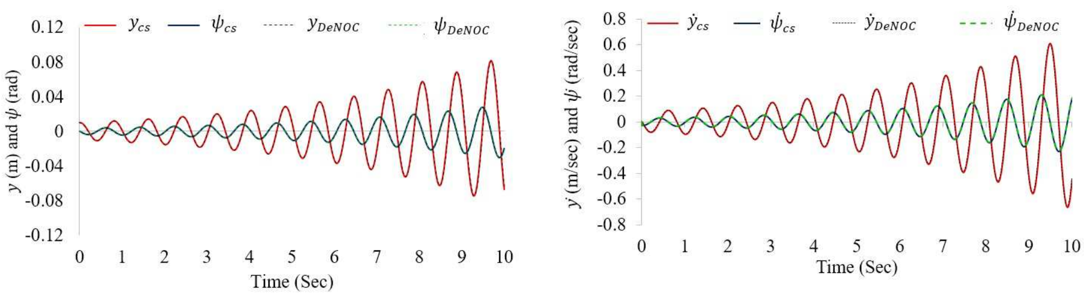
Where, $M = diag[M_1 \ \ M_2 \dots M_n], W = diag[W_1 \ \ W_2 \dots W_n]$, $\boldsymbol{w} = [\boldsymbol{w}_1^T \ \ \boldsymbol{w}_2^T \dots \boldsymbol{w}_n^T]^T$ and $\boldsymbol{t} = [\boldsymbol{t}_1^T \ \ \boldsymbol{t}_2^T \dots \boldsymbol{t}_n^T]^T$ and n is the number of links. Using recursive relations presented in [1], the relation between Cartesian and independent generalized speeds ($\dot{\boldsymbol{\phi}}_i$) are derived as follows.

$$\dot{\boldsymbol{q}}_i = N\dot{\boldsymbol{\phi}}_i \tag{4}$$

N is called as Natural Orthogonal Compliment (NOC) matrix. The NOC matrix is the column space of constraint Jacobian matrix C, therefore, $N^T C = \mathbf{0}$. As a consequence, pre multiplying equation (3) with N^T eliminates the lagrange multipliers and result in minimum set of equations of motion corresponding to system DOF's. Upon applying this technique and rearranging, we get

$$I\ddot{\boldsymbol{\phi}}_i = \boldsymbol{\tau} \tag{5}$$

In equation (5), I is the Generalized Inertia Matrix (GIM), $\boldsymbol{\tau}$ contains the vector of generalized forces and Coriolis terms. The serial chain modeling was applied to derive equations of motion of a free wheelset moving at constant forward velocity (a special case). Small angle approximations were used and some results are presented as follows.



(a) (b)

Figure 2: (a) Response of sway (y_w) and yaw (ψ_w), (b) Response of sway rate ($\dot{y}_w$) and yaw rate($\dot{\psi}_w$)

Equations of motion were also derived using conventional methods used in the literature [4]. Response of the system - position and velocity states, using conventional method and the DeNOC formulation are plotted in Figure 2(a) and 2(b) respectively. It can be inferred that, as described in literature [4,5] - motion of a free wheelset is unstable. The serial chain modeling technique is extended to derive equations of motion bogie and carbody subsystems systematically.

References

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