Generalized Constraint Embedding for Closed-Loop Multibody System Dynamics

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Abstract

The relative motion between adjacent bodies in a multibody system is constrained by the hinge connecting the bodies. Options to handle such constraints are to treat them as *hinges*, or as *bilateral constraints* between the body pair. In the hinge approach, the constraint is eliminated from the equations of motion by using minimal coordinates to parameterize the relative motion between the body pair. On the other hand, the bilateral constraint approach allows full 6 degree of freedom relative motion between the bodies, and uses additional constraints and Lagrange multipliers to impose the constraints on them.

A popular method for solving the dynamics of closed-loop systems is the *fully-augmented (FA)* dynamics algorithm [1, 7, 5]. In this method, all bodies are treated as independent bodies, and the relative motion constraints are handled explicitly as bilateral constraints. The advantage of this approach is that the equations of motion are simple and easy to set up. The mass matrix of the associated tree system is block diagonal and constant. Sparse matrix solution techniques can be used to solve for Lagrange multiplier constraint forces. Disadvantages include the large number of generalized coordinates, the underlying DAE structure of the equations of motion, and the need to use error control techniques to manage constraint errors during a simulation.

An alternative method is the *tree-augmented (TA)* dynamics algorithm [6, 2]. In this method, a minimal set of the inter-body constraints are "cut" to convert the system into a tree-topology system. The inter-body constraints within the tree are treated as hinges parameterized with a minimal set of coordinates. The overall dynamics model formulation consists of the minimal-coordinate dynamics model for the tree system together with the minimal set of bilateral closure constraints for the tree system. The advantages of this method are that the number of generalized coordinates and explicit bilateral constraints is much smaller compared with the FA model. Though the tree system mass matrix now is dense and configuration dependent, structure-based low-order recursive tree-topology algorithms can be used to solve for the Lagrange multipliers. The underlying formulation remains a DAE, but the error control is only needed for the smaller set of bilateral constraints.

The third method that avoids DAEs altogether is the recently developed *constraint embedding (CE)* dynamics algorithm [3]. This technique uses the TA model as a starting point. However, TA model's bilateral constraints are eliminated by aggregating bodies affected by the bilateral constraint into compound bodies. The result system topology is once again a tree with only inter-body hinges and no bilateral constraints. The benefit of this approach is that the structure-based tree algorithms can be directly used to solve the dynamics, and this formulation results in an ODE instead of a DAE. Thus error control techniques are not required. This method however is more complex to implement, since the aggregated bodies now have configuration dependent geometry. While CE method shares the minimal coordinates attribute with projection dynamics techniques [1, 7], its advantage lies in the preservation of the system's tree topology that is necessary for the use of the structure-based tree algorithms. A comparison of the error performance and computational time across the methods shows the CE method as having the best performance, the FA method significantly worse performance, with the TA method being in the middle [4]. These comparisons show that by taking advantage of structure-based low-order algorithms, the added complexity of the hinge-based methods is well rewarded by significantly superior performance over constraint-based methods.

While both the TA and CE methods both build on recursive minimal coordinate approaches, the analysis and and algorithms are very different. While the CE algorithm is undoubtedly superior for systems with small loops - and especially ones where the loop kinematics has analytical solution. However when the loops become larger, or more complex (eg. meshes), the benefits of the CE approach over the TA approach decrease. For systems involving both small and large loops, the analyst is left with the unsatisfactory situation of choosing between either the TA or CE approaches.

The main contribution of this paper is the development of a *generalized CE method*. In the generalized method, the TA and CE approaches can be applied to different loops simultaneously in the multibody system. This allows the user to judiciously choose the better of the TA or CE approaches for each of the loops in the system. For smaller loops the CE approach can be used to create compound bodies, while for larger loops the TA approach can be used to apply sufficient cuts to remove the loops. The generalized CE method continues to take advantage of minimal coordinate recursive algorithms for keeping computational costs low.
As a key step towards developing this generalized CE method, we also prove the mathematical equivalence of the TA and CE approaches. This analysis approach to showing this equivalence provides the building blocks for the development of the generalized CE method.

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