Application of an iterative rheo-linear frequency domain solver to unbalance effects of transmission systems

Klaus-Dieter Bauer*, **Josef Haslinger***, **Günter Offner#**

* Radon Institute for Computational and Applied Mathematics
Austrian Academy of Sciences and MathConsult GmbH
Altenberger Str. 69, 4040 Linz, Austria
kdbauer,josef.haslinger@mathconsult.co.at

Advanced Simulation Technologies
AVL List GmbH
Hans-List-Platz 1,
8020 Graz, Austria
guenter.offner@avl.com

Abstract

In early design phases of transmission systems, fast calculation of dynamic properties is essential for studying a wide range of system configurations. When modeled as a multi-body system, consisting of mechanical components (bodies), discretized into sets of nodes, and force-elements (joints) representing coupling between those bodies such as gears and bearings, the mathematical model comprises a large number of degrees of freedom. At high rotational speeds, standard transient time integration based on the BDF method as described in [1] has to be performed over hundreds of cycles to achieve convergence towards steady-state operational conditions, as initial transient oscillations decay slowly. Such computations can take several hours for complex systems even on modern hardware.

A faster solution can be achieved by solving a linearized equation of motion in frequency domain: Let $z_0(t)$ be a known, approximated trajectory of the system ("reference trajectory"), optionally refined using kinetostatic relaxation [2]. Let $q(t) \in \mathbb{R}^{N_{\mathrm{DOF}}}$ be a vector, describing deviations from the reference trajectory in N_{DOF} degrees of freedom ("DOFs"). Then the equations of motion can be linearized into a form

$$M\ddot{q}(t) + D\dot{q}(t) + K q(t) = f(t), \qquad (1)$$

where M, D, K ("dynamic matrices") are $N_{\mathrm{DOF}} \times N_{\mathrm{DOF}}$ matrices, describing inertia, damping and stiffness effects respectively, and $f(t)$ a function describing external loads, inertia-forces, as well as stiffness and damping forces occurring along the reference trajectory from the internal stiffness and damping of elastic bodies as well as from joints. The matrices M, D, K have block structure, associated with bodies and joints: Entries M_{ij}, D_{ij}, K_{ij}, where i, j denote DOFs of the same body, describe inertia, internal friction and elastic behavior of that body and form on-diagonal square blocks; Entries, for which i, j are associated with different bodies, result from linearization of the joint forces and form off-diagonal rectangular blocks for each pair of bodies.

When the matrices M, D, K are time-independent, and functions are decomposed into a suitable discrete set of harmonic components according to $f(t) = \sum_n f_n e^{i\omega_n t}$ with $\omega_n = n\,\omega_1$, a frequency domain representation

$$\left(-\omega_n^2 M + i\,\omega_n D + K\right) q_n = f_n \qquad (2)$$

is obtained. For N_{freq} frequency components, we thus have N_{freq} equation systems in N_{DOF} variables each, solvable in $O(N_{\mathrm{freq}} N_{\mathrm{DOF}}^2)$, reducing the computation time for slowly converging systems by as many as two orders of magnitude.

However, only sufficiently simple models result in time-independent linearization coefficients; Gear-meshing effects for instance result in a variaton of stiffness and damping matrix along the reference trajectory. The time-dependence however results in equations of motion of the form

$$\sum_n \left(-\omega_n^2 M_{m-n} + i\,\omega_n D_{m-n} + K_{m-n}\right) q_n = f_m, \qquad (3)$$

resulting in a single equation system in $N_{\mathrm{DOF}} \times N_{\mathrm{freq}}$ variables. Naive solution would therefore increase the computational effort by $O(N_{\mathrm{freq}})$ to $O(N_{\mathrm{freq}}^2 N_{\mathrm{DOF}}^2)$. With the frequencies ω_n often numbering in the hundreds or thousands, such an increase would eliminate any performance advantage gained from switching from time to frequency domain.

In [3] we have presented an iterative approach based on perturbation theory, that reduces the problem back to solving independent equation systems. Applied to gear-meshing effects, satisfactory

convergence was reached after just 3 iterations. In this contribution, we study the use of this approach for accounting for unbalance effects: Let the model contain two or more bodies, for which the reference trajectory $z_0(t)$ consists of uniform rotations around separate axes for each body, with dynamic matrices M, D, K not being invariant under these rotations. In a global inertial frame of reference, the matrices will then be time-dependent.

In the floating frame of reference formulation [4], separate coordinate systems are chosen for each body, such that matrix entries describing coupling between nodes of the same body become time-independent. However, even a joint representing a linear spring between nodes of different bodies would result in time-dependent entries.

This knowledge does however allow us to formulate the time-dependence of the on-diagonal per-body blocks of the dynamic matrices in terms of the rotation matrix $R(t)$ associated with the axial rotation of the body in the reference trajectory $z_0(t)$: Let $u_i(t) \in \mathbb{R}^3$ be the translatory displacement of node i and $\varphi_i(t) \in \mathbb{R}^3$ describe the rotational displacement of node i as a vector of small axial rotations $\varphi_i = (\varphi_{i,x}, \varphi_{i,y}, \varphi_{i,z})$. The displacement vector then has the shape

$$q(t) = (u_i(t), \varphi_i(t), \ldots, u_{N_{\text{nodes}}}(t), \varphi_{N_{\text{nodes}}}(t)) \tag{4}$$

with $N_{\text{DOF}} = 6 N_{\text{nodes}}$. Let nodes $i \in \{1, \ldots, N_{\text{B}}\}$ be the DOFs associated with the first body and $K_{\text{B}} \in \mathbb{R}^{N_{\text{B}} \times N_{\text{B}}}$ be the constant elastic matrix of the body in the rotating frame of reference. Let $R_{\text{B}}(t)$ be a rotation matrix describing the uniform axial rotation of the body from the reference trajectory, and let

$$\tilde{R}_{\text{B}}(t) = \begin{pmatrix} R_{\text{B}}(t) & 0 & \cdots & 0 & 0 \\ 0 & R_{\text{B}}(t) & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & 0 & 0 \\ 0 & 0 & \cdots & R_{\text{B}}(t) & 0 \\ 0 & 0 & \cdots & 0 & R_{\text{B}}(t) \end{pmatrix} \in \mathbb{R}^{N_{\text{B}} \times N_{\text{B}}}. \tag{5}$$

Then the sub-matrix of K associated with the first body can be written as

$$\left(K_{ij}(t) \right)_{i,j=1,\ldots,N_{\text{B}}} = \tilde{R}_{\text{B}}(t) \cdot K_{\text{B}} \cdot \tilde{R}_{\text{B}}^T(t).$$

Similar relations can be formulated for the matrices M, D and for each of the bodies, yielding an explizit analytical form of the time-dependence of the dynamic matrices.

We study evaluation of the resulting frequency-domain equations of motion using the iterative algorithm presented in [3] on the example of a simple transmission system, consisting of multiple shafts with unbalance, i.e. with time-dependent per-body dynamic matrices as described in (5).

References

[1] Drab, C. B.; Engl, H. W.; Haslinger, J.; Offner, G.; Pfau, R. U.; Zulehner, W.: Dynamic simulation of crankshaft multibody systems. Multibody System Dynamics, Vol. 22, pp. 133–144, 2009.

[2] Lorenz, N.; Offner, G.; Colla, A.: Preloaded Initial State Evaluation of Flexible Multibody Systems. ECCOMAS Thematic Conference: Multibody Dynamics, Zagreb, 2013.

[3] Bauer, K. D.; Haslinger, J.; Offner., G.: Approaches for going beyond linear frequency domain powertrain simulations. 21st ECMI Conference on Industrial and Applied Mathematics, 2021.

[4] Offner, G.: Modelling of condensed flexible bodies considering non-linear inertia effects resulting from gross motion. Proceedings of the Institution of Mechanical Engineers, Part K: Journal of Multi-body Dynamics, Vol. 225, pp. 204-219, 2011.