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Dynamics of moving load on simply-supported beam using sprung-mass model

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Abstract

One of the classical problems in structural dynamics is the dynamics of moving loads over the beam which changes with respect to space and time. Most of the researchers [1], [2], [3], [4] (to name a few) paid attention to the idealization of traveling forces on a beam using moving force (figure 1 (a)) or moving mass model. Such models do not consider the interaction between the beam and the moving vehicle. The simplest model to account for the bouncing or suspension action of the moving vehicle is the sprung mass model. Such a model is used by many researchers [5], [6], and [7] highlighting the effect of interaction between two sub-systems (vehicle and beam). A comparison between the above-mentioned dynamic load models has been presented in this paper.

The dynamic interaction between moving vehicle and bridge have been analyzed using mode-superposition method. The moving vehicle is idealized as sprung-mass model which consists of a concentrated mass 'm_v' supported by a spring of stiffness 'k_v'. A uniform simply supported beam of Euler-Bernoulli type have been addressed under the effect of the moving vehicle at speed 'v' as shown in Figure.1 (a). No such pavement irregularity or damping property of the bridge/suspension were considered.

The vertical vibration of the moving vehicle-bridge system can be expressed by following equations of motion

$$Mu + EIu''''^{\ddot{i}} = p(x,t) \tag{1}$$

$$m_v\ddot{q}_v + k_v q_v = k_v u|x = vt \tag{2}$$

where 'M' denotes the mass per unit length, E is the elastic modulus, I is the moment of inertia of the beam, $u(x,t)$ and 'q_v' is the vertical displacement of the bridge and vehicle. The applied force $p(x,t)$ acting on the bridge through point of contact point at position 'vt', moves with the moving vehicle given by expression:

$$p(x,t) = f_c(t)\delta(x - vt) \tag{3}$$

where δ is the Dirac delta function evaluated at the contact point, x=vt, and the contact force f_c is evaluated as sum of the vehicle weight and the elastic force of the corresponding suspension system, i.e. $f_c(t)=-m_v g+k_v(q_v-u|_{x=vt})$, where '$g$' represents acceleration due to gravity.

Using Modal superposition method, the solution of Equation (1) can be expressed in terms of mode shapes, $\varphi_n(x)$ and modal coordinates, $q_{bn}(t)$ as

$$u(x,t) = \sum_n \varphi_n(x) q_{bn}(t) = \sum_n [sin\frac{n\pi x}{L} q_{bn}] \tag{4}$$

The coupled vehicle-track interaction can be expressed as:

$$\begin{pmatrix}\ddot{q}_b\\ \ddot{q}_v\end{pmatrix} + \begin{bmatrix}2\omega_v^2 sin^2\frac{\pi vt}{L} + \omega_{bn}^2 & -2\omega_v^2\frac{m_v}{ML}sin\frac{\pi vt}{L}\\ \omega_v^2\frac{m_v}{ML}sin\frac{\pi vt}{L} & \omega_v^2\end{bmatrix}\begin{pmatrix}q_b\\ q_v\end{pmatrix} = \begin{matrix}-2\omega_v^2\frac{m_v}{ML}sin\frac{\pi vt}{L}\\ 0\end{matrix} \tag{5}$$

where ω_{bn} is the vibrational frequency of the bridge corresponding to the n^{th} mode and ω_v is the vibrational frequency of the vehicle given as

$$\omega_{bn} = \frac{n^2\pi^2}{L^2}\sqrt{\frac{EI}{M}};\ \omega_v = \sqrt{\frac{k_v}{m_v}} \tag{6}$$

By solving Equation (5) in MATLAB, the vertical displacement at midpoint of the beam (Figure. 1 (b)), vertical acceleration of the beam (Figure.1 (c)), and vertical acceleration of the sprung mass (Figure. 1

(d)) confirms the confidence about the presented method with the reported literature [5]. Further, a comparison between moving load model and sprung-mass model has been presented in Figure 1 (b) and (c) which shows that moving sprung mass model considers interaction effects, and results in displacement of beam a little on higher side. Additionally, the response of acceleration of sprung mass has been taken as measure of the passengers riding comfort which moving load model cannot incorporate. Therefore, as far as the riding comfort criteria is concerned, it is essential to consider the influence of vehicle parameters into the analysis.

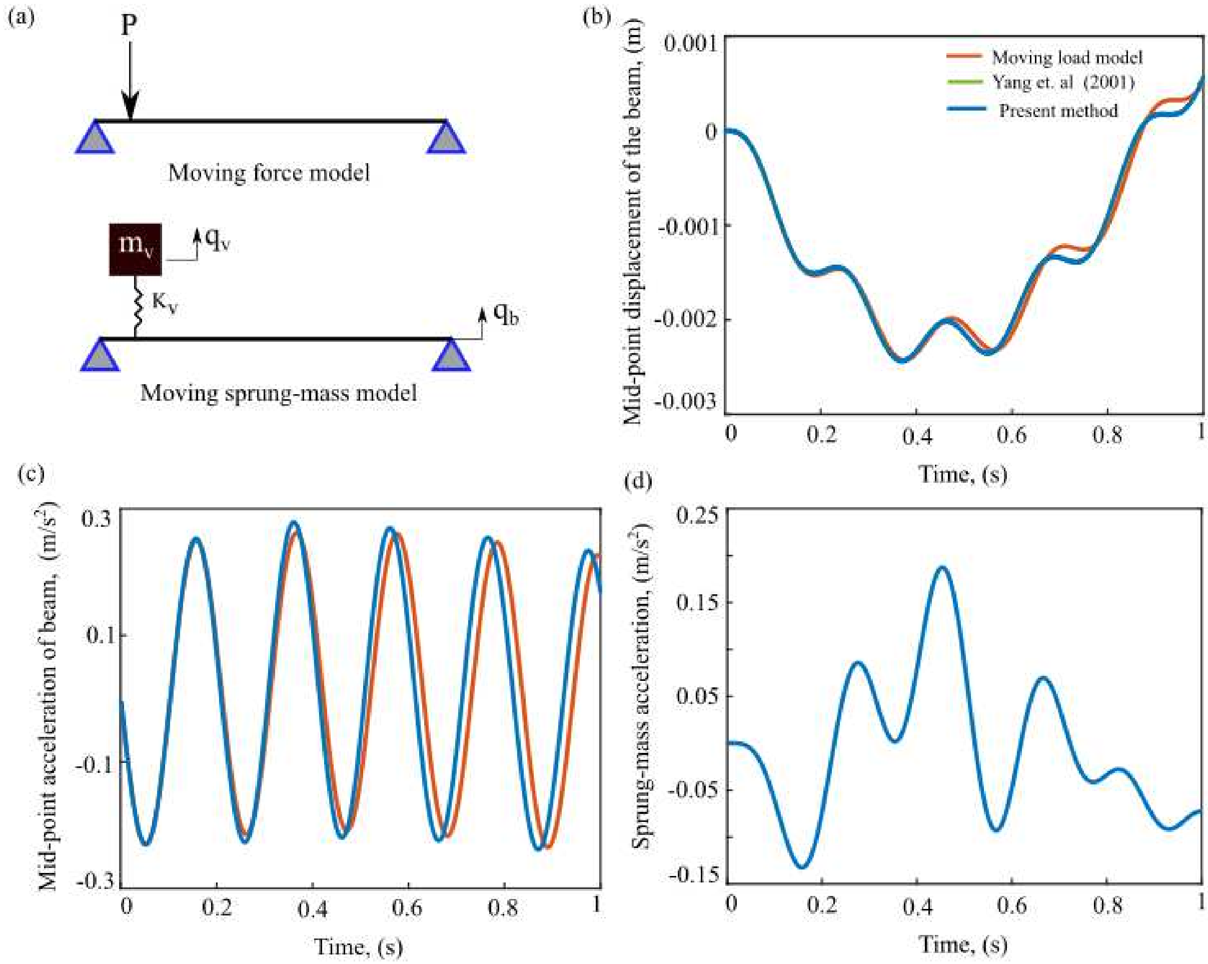

Figure 1: (a) Idealisation of moving load and sprung-mass model over simply-supported beam; (b) Mid-point displacement of the beam; (c) Mid-point acceleration of the beam; and (d) Acceleration response of the sprung mass system.

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