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Command Shaped Trajectory Tracking Control for a Two-link Flexible Manipulator

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Abstract: Flexible manipulators are subjected to structural vibrations that need to be suppressed. This work presents a hybrid control scheme to suppress these vibrations for a two-link flexible manipulator while tracking curved and linear path (semicircle in closed form) trajectory with high speed. The dynamic modeling for this purpose is performed using assumed mode method (AMM) where equations of motion are derived using decoupled natural orthogonal complement (DeNOC) matrices. In order to ensure the accuracy of developed mathematical model, the dynamic model is validated in both frequency and time domain analysis. Thereafter, hybrid controller is developed which is based on command shaping in feedforward mode and PD control in the feedback mode. The desired hub angles are calculated using inverse kinematics approach. The results include the tip deflections of links and tracking error around the desired trajectory.

Keywords: Flexible Manipulators, trajectory tracking, assumed mode method, command shaping, DeNOC matrices.

Introduction: Flexible manipulators, compared to rigid manipulators, are light in weight, consumes less power, provides high speed operations and have high payload-to-weight ratio but are subjected to structural vibration [1]. Flexible manipulators, generally have infinite degrees of freedom but for simulation purpose, these manipulators are modelled as finite degrees of freedom system. In this work, the dynamic equations are obtained using Assumed mode method (AMM) and Decoupled natural orthogonal complement (DeNOC) matrices [2]. For trajectory tracking, simple proportional derivative (PD) controller used while command shaping [3] is used for suppressing the vibrations.

Dynamic Model: For dynamics purpose, links deformations are modelled by assuming 2 modes in one transverse direction, neglecting other vibration. The dynamic model of the flexible manipulators in general form can be written as.

$$\boldsymbol{I}\ddot{\boldsymbol{q}} + \boldsymbol{h} = \boldsymbol{\tau} \tag{1}$$

Here $\boldsymbol{I}$ is inertia matrix and $\boldsymbol{h}$ is vector of convective inertia terms, while $\boldsymbol{\tau}$ is generalized force vector having terms related to actuator effort ($\boldsymbol{\tau}^E$), stiffness ($\boldsymbol{\tau}^s$) and gravity ($\boldsymbol{\tau}^g$). For validation of dynamic model, links length is kept as 1m while $EI_{zz} = 100Nm^2$, where E is young's modulus of elasticity and I_{zz} is area moment of inertia in bending. The free fall simulation is done under gravity, where initial angles are kept as -90^0 and 5^0 for first link and second link, respectively. The hub angles for first link and second link are shown in Fig. 1a and 1b, respectively. The results are found to be in close agreement with the published literature [2].

Trajectory Tracking: Simulation is done for semicircular trajectory tracking for two link flexible manipulator. The links are of length 0.5 m, made of material with density 7850 $\mathrm{Kg/m^3}$, and modulus of elasticity equals to 210 GPa. The cross section of links is in form of rectangle with thickness 2 mm and width 27 mm. For command shaping calculation, natural frequency and damping ratio are determined as 4.49 Hz and 0.025, respectively, at the extreme point using MATLAB.

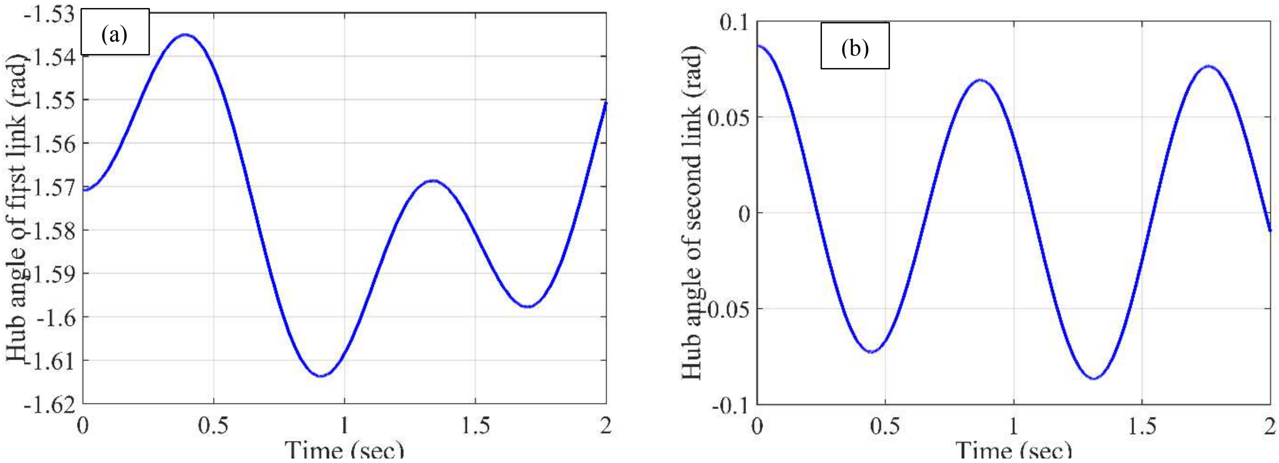

Fig. 1: Hub angle for (a) first link (b) second link

There is slight change of 0.24 Hz in the natural frequency when the configuration changed while tracking, which has been considered in calculations. The total time duration planned for whole trajectory is 3 seconds which is divided into 2 seconds and 1 second for semicircle and linear path, respectively. Tip acceleration is kept in bang-bang fashion to get rest to rest motion as shown in Fig. 2a (for curved part of trajectory). The tracking of semicircular profile can be seen in the Fig. 2b. The maximum errors between desired and obtained trajectory are 6.327mm and 8.217mm which has been reduced to 5.214 mm and 3.057 mm for curved and linear part, respectively, of the trajectory using command shaping.

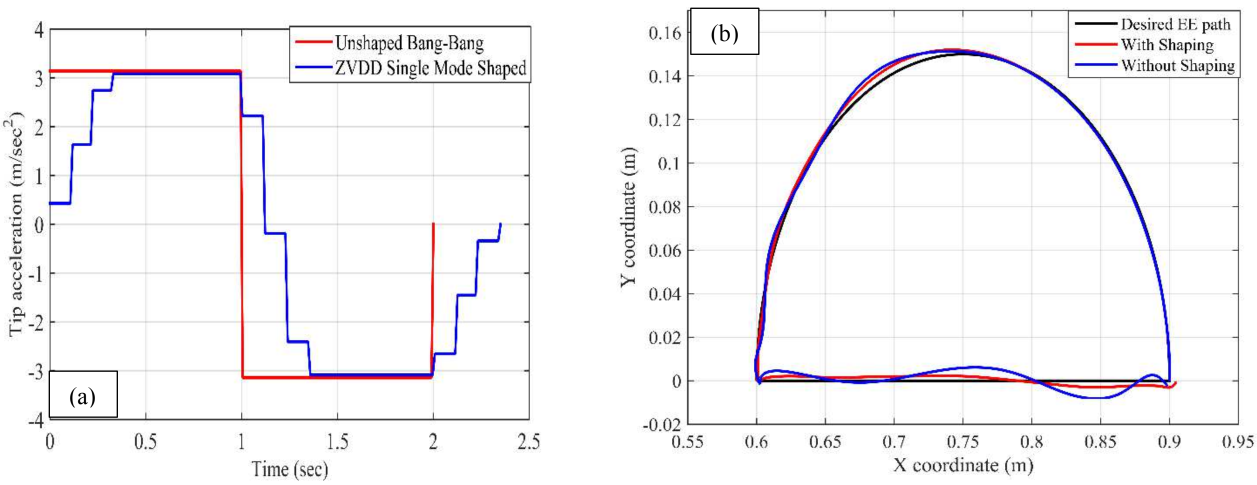

Fig. 2: (a) Tip acceleration for curved part of trajectory (b) End effector path for with and without shaping

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