Formation of a wide low frequency attenuation bandgap in a mass-in-mass frictional metamaterial

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Abstract

The main focus of this paper is to propose a linear complementary based algorithm along with the use of Euler's discretization to solve the dynamics of frictional systems. A mass-in-mass 2-DOF system with internal frictional contact has been taken into consideration and the proposed methodology has been applied to study the dynamic response of that system. The same unit cell is then used to form a metamaterial chain and the attenuating potential of the same has been studied.

At the interface of two masses in contact, Coulomb's friction law has been used to model the friction. The statement of Coulomb's friction law can be expressed in mathematical form as [3]:

$$\begin{aligned} |\lambda_T| < \mu\lambda_N &\implies \dot{g_T} = 0 \rightarrow \text{sticking} \\ \lambda_T = \mu\lambda_N &\implies \dot{g_T} < 0 \rightarrow \text{backward slipping} \\ \lambda_T = -\mu\lambda_N &\implies \dot{g_T} > 0 \rightarrow \text{forward slipping} \end{aligned} \tag{1}$$

where, λ_T defines the tangential friction force, λ_N defines the normal reaction force, μ defines the co-efficient of friction, g_T defines the gap between two points located in the two different surfaces in the transverse direction; hence, its time derivative $\dot{g_T}$ represents the relative velocity.

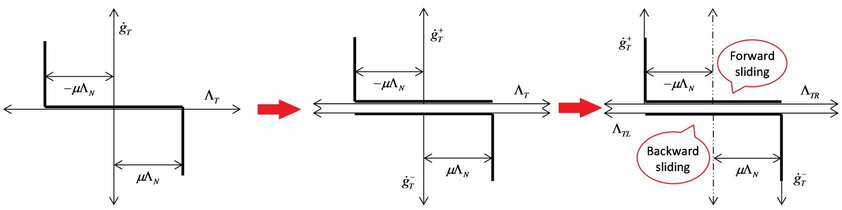

Figure 1: LCP formulation from Coulomb's friction

The linear complementary form of Equation 1 is:

$$\begin{aligned} \lambda_{TR}^T \dot{g}_T^+ &= 0 \\ \lambda_{TL}^T \dot{g}_T^- &= 0 \end{aligned} \tag{2}$$

where, λ_{TR}, λ_{TL} and $\dot{g}_T^-$, $\dot{g}_T^+$ are the required quantities to perform the coordinate transformation as shown in Figure 1 such that:

$$\left.\begin{aligned} \lambda_{TR} &= \mu\lambda_N + \lambda_T \\ \lambda_{TL} &= \mu\lambda_N - \lambda_T \end{aligned}\right\} \rightarrow \left\{\begin{aligned} \lambda_{TR} - \lambda_{TL} &= 2\lambda_T \\ \lambda_{TR} + \lambda_{TL} &= 2\mu\lambda_N \end{aligned}\right. \tag{3a}$$

$$\dot{g_T} = \dot{g_T}^+ - \dot{g_T}^- \rightarrow \Delta X = (\dot{g_T}^+ - \dot{g_T}^-)\Delta t \tag{3b}$$

Here, $\dot{g}_T^+ = \frac{|\dot{g}_T| + \dot{g}_T}{2}$, if $\dot{g}_T \geq 0, \dot{g}_T^- = \frac{|\dot{g}_T| - \dot{g}_T}{2}$, if $\dot{g}_T \leq 0$.The equation of motion of a dynamic system having normal and tangential forces can be written in the acceleration level as[2]:

$$M\ddot{u} - h - W_N\lambda_N - W_T\lambda_T = 0 \tag{4}$$

Applying Euler's discretization to Equation 4, we get:

$$\begin{aligned} M\Delta q - h\Delta t - W_N\Lambda_N - W_T\Lambda_T &= 0 \\ \Delta q &= M^{-1}h\Delta t + M^{-1}W_N\Lambda_N + M^{-1}W_T\Lambda_T \\ \Delta u &= (q + \Delta q)\Delta t \end{aligned} \tag{5}$$

where $q = \dot{u}$, Λ_i=$\lambda_i\Delta t$. The first order Taylor series expansion of normal gap g_N and relative velocity in tangential direction $\dot{g_T}$ yields:

$$g_N^e = g_N + \Delta g_N(u,t) = g_N + \underbrace{\frac{\partial g_N}{\partial u}}_{W_N^T} \Delta u + \underbrace{\frac{\partial g_N}{\partial t}}_{\tilde{\omega}_N} \Delta t \tag{6a}$$

$$\dot{g_T}^e = \dot{g_T} + \Delta \dot{g_T}(u,\dot{u},t) = \dot{g_T} + \underbrace{\frac{\partial \dot{g_T}}{\partial \dot{u}}}_{W_T^T} \Delta q + \underbrace{\frac{\partial \dot{g_T}}{\partial u}}_{\dot{W}_T^T} \Delta u + \underbrace{\frac{\partial \dot{g_T}}{\partial t}}_{\dot{\tilde{\omega}}_T} \Delta t \tag{6b}$$

Substituting the values of Δq and Δu from Equation 5 into Equation 6a and Equation 6b, and rearranging the terms we get the final LCP equation as:

$$\begin{Bmatrix} g_N^e \\ \dot{g}_T^{e+}\Delta t \\ \Lambda_{TL}\Delta t \end{Bmatrix} = \begin{bmatrix} G_{NN} - \mu G_{NT} & G_{NT} & 0 \\ G_{TN} - \mu G_{TT} & G_{TT} & I \\ 2\mu & -I & 0 \end{bmatrix} \begin{Bmatrix} \Lambda_N \Delta t \\ \Lambda_{TR}\Delta t \\ \dot{g}_T^{e-}\Delta t \end{Bmatrix} + \begin{Bmatrix} C_N \\ C_T \Delta t \\ 0 \end{Bmatrix} \tag{7}$$

where $G_{NN} = W_N^T M^{-1} W_N$, $G_{NT} = W_N^T M^{-1} W_T$, $G_{TN} = (W_T^T + \dot{W}_T^T \Delta t) M^{-1} W_N$, $G_{TT} = (W_T^T + \dot{W}_T^T \Delta t) M^{-1} W_T$, $C_N = W_N^T (q + M^{-1} h \Delta t)\Delta t + \tilde{\omega}_N \Delta t + g_N$ and $C_T = (G_T h \Delta t + \dot{W}_T^T q \Delta t + \dot{\tilde{\omega}}_T \Delta t + \dot{g}_T)$.

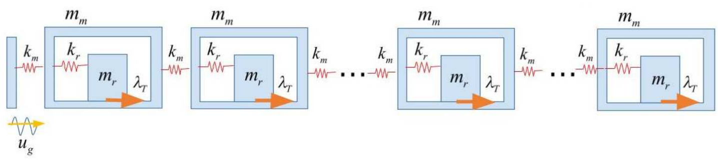

Figure 2: A metamaterial chain comprising of finite number of mass-in-mass frictional unit cells

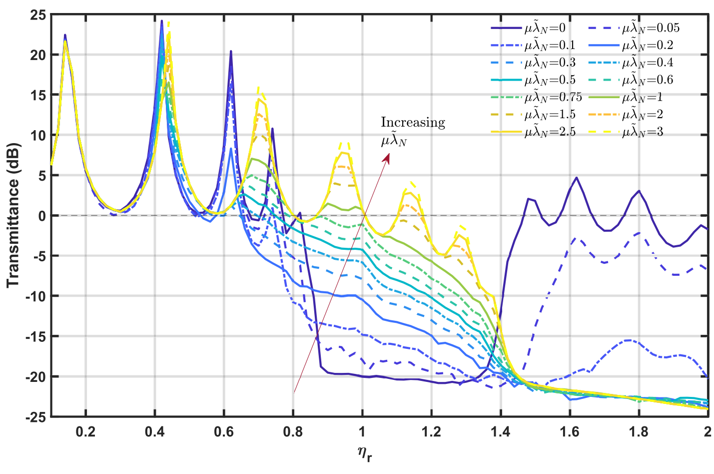

Figure 3: Transmittance with respect to frequency ratio (η_r) plot for the 7^{th} unit in the metamaterial chain for varying non dimensional maximum frictional force ($\mu\tilde{\lambda}_N$)

The developed scheme has been implemented to solve the dynamics of a 2-DOF mass-in-mass frictional unit cell. The same unit cell has been used to form a metamaterial chain comprising of finite number of unit cells as shown in Figure 2. Transmittance spectrum has been plotted for the 7^{th} unit cell of the metamaterial chain comprising of seven unit cells, and given in Figure 3.
From the transmittance spectrum, it is observed that the attenuation band gap in the lower frequency range is wider when the frictional force between the two masses is upto $\mu\tilde{\lambda}_N = 0.6$, compared to the case when frictional force is zero[1]. Further on increasing the frictional force, each unit behaves like a 1-DOF unit, rather than a mass-in-mass unit, and the first cut-off frequency shifts towards higher frequency range. Therefore, it is concluded that mid-range values of frictional force $\mu\tilde{\lambda}_N$ are most suitable for attaining wider low frequency attenuation band gap. Moreover,for $\mu\tilde{\lambda}_N = 0.3$, the low frequency attenuation band gap is the widest.

References

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