Flexible Rotating Multibody Analysis Using Extended NPFEM for Non-Equatorial Space Elevator

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Introduction

A space elevator is a next-generation space transportation system using climbers moving on a tether deployed from the geostationary orbit (GEO). In particular, a non-equatorial space elevator has recently attracted attention owing to its advantages, such as extending the construction range and avoiding collisions with spacecraft in the GEO [1]. Past studies have used low-fidelity rigid-body or spring-mass models [2]. This study presents a multibody modeling method for non-equatorial space elevators using a nodal-position finite element method (NPFEM [3]) extended to a rotational coordinate system. The NPFEM is a geometrically nonlinear finite element method that accounts for the coupling effect between rigid-body rotation and elastic deformation of the element. Conventional NPFEMs have only been used with inertial coordinate systems. This study presents the NPFEM formulated in a non-inertial coordinate system and obtains the inertial forces and Jacobian matrices. Additionally, a non-equatorial space elevator in three-dimensional space was analyzed based on the method.

Methods

The coordinate system and model used in this study are illustrated in Fig. 1(a). The X–Y plane of the three-dimensional Cartesian coordinate system denotes the equatorial plane, and the origin O denotes the center of the Earth. Notably, the coordinate system is a noninertial coordinate system rotating around the Z-axis with an Earth's angular velocity ω_{E} ($\mathbf{\Omega}_{\mathrm{E}} \equiv \begin{bmatrix} 0 & 0 & \omega_{\mathrm{E}} \end{bmatrix}^{\mathrm{T}}$). A space elevator comprises a tether, climber, and counterweight. Both the climber and the counterweight are modeled as lumped masses. The flexible tether that rotates at the same angular velocity as the Earth is modeled using an NPFEM extended to a rotational coordinate system. One end of the tether connecting to the Earth is constrained by a ball joint as an anchor, whereas the end on the counterweight side is free. Only the gravitational and inertial forces act as external forces in this coordinate system. The Earth was assumed to be a perfect sphere.

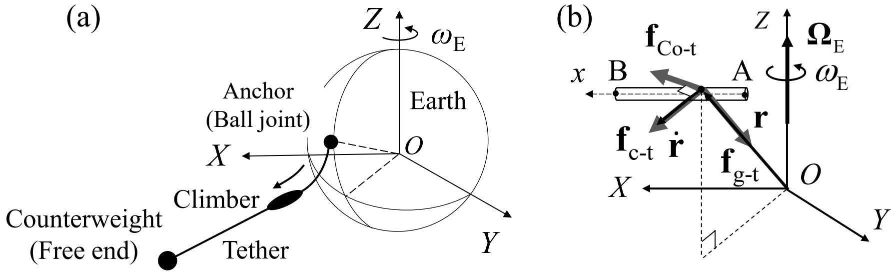

Fig. 1 (a) Components of non-equatorial space elevator and coordinate system (b) Gravitational force $\mathbf{f}_{\text{g-t}}$, centrifugal force vector $\mathbf{f}_{\text{c-t}}$, and Coriolis force vector $\mathbf{f}_{\text{Co-t}}$ acting on arbitrary point in tether element

The position vector $\mathbf{r}$ at an arbitrary point on the x-axis of the tether element can be expressed as

$$\mathbf{r} \equiv \begin{bmatrix} r_1 & r_2 & r_3 \end{bmatrix}^{\mathrm{T}} = \mathbf{S}\mathbf{q},\ \mathbf{S} \equiv \begin{bmatrix} (1 - x/l_0)\mathbf{I}_{3\times3} & \xi\mathbf{I}_{3\times3} \end{bmatrix},\ \mathbf{q} \equiv \begin{bmatrix} (\mathbf{r}^{\mathrm{A}})^{\mathrm{T}} & (\mathbf{r}^{\mathrm{B}})^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}, \tag{1}$$

where $\mathbf{r}^{\mathrm{A}}$ and $\mathbf{r}^{\mathrm{B}}$ represent the position vector of the nodes at both ends of the element in the absolute coordinate, l_0 represents the natural length of the tether element. The generalized external force vector of the tether element $\mathbf{Q}_{\text{ext-t}}$ can be derived based on the principle of virtual work as

$$\begin{gathered}\mathbf{Q}_{\text{ext-t}} \equiv \int_0^{l_0} \mathbf{S}^{\mathrm{T}}\left(\mathbf{f}_{\text{g-t}} + \mathbf{f}_{\text{c-t}} + \mathbf{f}_{\text{Co-t}}\right) A\, dx, \\ \mathbf{f}_{\text{g-t}} \equiv -\mu\rho \frac{\mathbf{r}}{|\mathbf{r}|^3},\ \mathbf{f}_{\text{c-t}} \equiv \rho\omega_{\mathrm{E}}^2 \mathbf{r}\Big|_{r_3^{\mathrm{A}}, r_3^{\mathrm{B}}=0}^{\mathrm{T}},\ \mathbf{f}_{\text{Co-t}} \equiv 2\rho\, \dot{\mathbf{r}} \times \mathbf{\Omega}_{\mathrm{E}},\end{gathered} \tag{2}$$

where ρ denotes the material density of the tether and μ denotes the geocentric constant. A denotes the cross-sectional element of each tether element. A differs in each element since the tether is assumed the tapered shape. Superposing each elemental matrix provides the equation of motion for the entire tether system. The climber motion is formulated referring to the method proposed by Sun *et al.* [4]. Superposing the mass matrix and the generalized force vector of the climber and the counterweight to those of all tether elements provides the equation of motion of the space elevator system. The partial differentiation of the generalized force vector of the non-equatorial space elevator with the variable $\mathbf{q}$ yields the Jacobian matrix used for static analysis.

Results and conclusions

The proposed model provided the equilibrium position 3,300 km higher in the X-direction and 50 km higher in the Z-direction in the static analysis. Consequently, as depicted in Fig. 2, the proposed model exhibited a more significant dynamic response than the conventional model. It is because the proposed model considers the geometric nonlinearity of the tether. In conclusion, this study established the flexible multibody model for non-equatorial space elevators. This model considers the geometric nonlinearities and the coupling effect of rigid-body motion and elastic deformation of the tether. This study revealed that the non-equatorial space elevator could have larger displacement than conventional models [2].

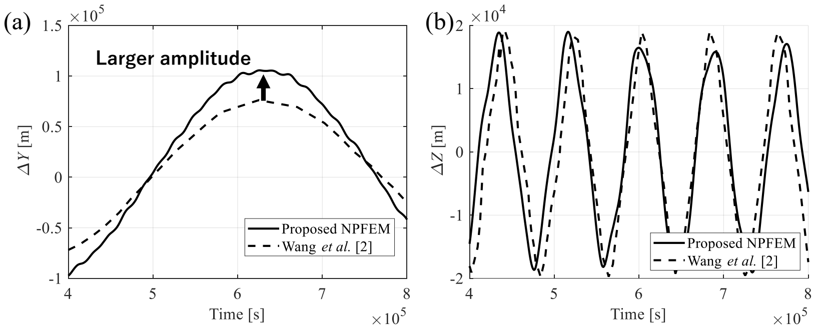

Fig. 2 Comparison of displacements of tether free end at climber ascent (a) Y-direction, (b) Z-direction

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