A Unified Framework for Linearly-Elastic Flexible Multibody System Dynamics Formulations

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Abstract

Flexible multibody (MB) system dynamics refers to computational strategies used to determine time histories of motion, deformation, etc. of interconnected components undergoing large overall motion due to applied forces, constraints, contact, and initial conditions. Linearly-elastic flexible MB simulations are oftentimes sufficient and are usually based on a floating frame approach, where a body-fixed frame follows the rigid body (RB) motion and the deformation is analyzed within this local frame; this description assumes the presence of large RB translations and rotations, small flexible deformations and strains within each body, and that the bodies obey a linear constitute law.

Recently it has been proposed [5, 6, 7] and even more recently shown more rigorously [10] that linearly-elastic MB systems discretized via isoparametric finite elements (FEs) may be described fully by the constant mass matrix $\overline{\boldsymbol{M}}$ and stiffness matrix $\overline{\boldsymbol{K}}$ from the underlying linear FE model as well as the corresponding nodal quantities. Hence, the kinetic energy T and strain energy U may be expressed as

$$T = \frac{1}{2}\dot{\boldsymbol{r}}^{\mathrm{T}}\overline{\boldsymbol{M}}\dot{\boldsymbol{r}}, \tag{1}$$

$$U = \frac{1}{2}\overline{\boldsymbol{c}}_{\mathrm{f}}^{\mathrm{T}}\overline{\boldsymbol{K}}\overline{\boldsymbol{c}}_{\mathrm{f}}, \tag{2}$$

respectively, where $\boldsymbol{r}$ denotes the global total nodal positions, $\overline{\boldsymbol{c}}_{\mathrm{f}}$ the local flexible nodal displacements, and $\dot{\bullet}$ the time derivative. Note that overlined quantities $\overline{\bullet}$ are expressed in the local frame in contrast to their global counterparts.

In [10] the authors also showed that the equations of motion (EOMs) for these systems may be described by formulation-independent EOMs given by

$$\boldsymbol{L}^{\mathrm{T}}\overline{\boldsymbol{M}}\boldsymbol{L}\ddot{\boldsymbol{q}} + \boldsymbol{L}^{\mathrm{T}}\overline{\boldsymbol{M}}\dot{\boldsymbol{L}}\dot{\boldsymbol{q}} + \boldsymbol{P}^{\mathrm{T}}\overline{\boldsymbol{K}}\overline{\boldsymbol{c}}_{\mathrm{f}}(\boldsymbol{q}) + \boldsymbol{J}^{\mathrm{T}}\boldsymbol{\lambda} = \boldsymbol{L}^{\mathrm{T}}\boldsymbol{f} \tag{3}$$

where

$$\boldsymbol{L} = \frac{\partial \boldsymbol{r}}{\partial \boldsymbol{q}}, \tag{4}$$

$$\boldsymbol{P} = \frac{\partial \overline{\boldsymbol{c}}_{\mathrm{f}}}{\partial \boldsymbol{q}}, \tag{5}$$

$$\tag{6}$$

and

$$\boldsymbol{J} = \frac{\partial \boldsymbol{g}}{\partial \boldsymbol{q}}, \tag{7}$$

are Jacobian matrices given by the partial derivatives of $\boldsymbol{r}$, $\overline{\boldsymbol{c}}_{\mathrm{f}}$, and the constraint equations $\boldsymbol{g}(\boldsymbol{q},t) = \boldsymbol{0}$ with respect to (w.r.t.) the generalized degrees of freedom (DOFs) $\boldsymbol{q}$. Furthermore, $\boldsymbol{f}$ and $\boldsymbol{\lambda}$ represent the applied nodal forces and Lagrange multipliers, respectively. Hence, to define a linearly-elastic MB formulation the following steps are sufficient [10]:

1. Choose the DOFs $\boldsymbol{q}$ – this choice defines the formulation.

2. Define the coordinate mappings $\boldsymbol{r} = \boldsymbol{r}(\boldsymbol{q})$ and $\overline{\boldsymbol{c}}_{\mathrm{f}} = \overline{\boldsymbol{c}}_{\mathrm{f}}(\boldsymbol{q})$.

3. Calculate the Jacobians of the coordinate mappings, i.e., $\boldsymbol{L}$ and $\boldsymbol{P}$.

4. Calculate the time derivative of $\boldsymbol{L}$, i.e., $\dot{\boldsymbol{L}}$.

5. Perform the matrix multiplications to obtain the final EOMs.

These steps were outlined in [10] to derive the conventional inertia-shape-integral/ lumped-mass floating frame of reference formulation (FFRF) with and without modal reduction [3], the nodal-based FFRF with [8] and without [7] modal reduction, the absolute coordinate formulation (ACF) [1, 5], the generalized component mode synthesis (GCMS) [2, 6], and the flexible natural coordinate formulation (FNCF) [4]. Moreover, more recently, improved variants of ACF and GCMS have been proposed also within this framework [9].

This contribution, therefore, summarizes the derivation of the unified framework and presents an overview of the ingredients, see Eq. (3), necessary to deduce the EOMs of the aforementioned formulations within this framework, i.e., a summary of [9, 10].

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