Approaches for Feedforward Control of Flexible Multibody Systems Modeled by the ANCF

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Abstract

Current design trends as well as the demand for fast actuation and low power consumption result in the design of flexible multibody systems. While the flexibility is sometimes desired for example for human-structure interaction, it also results in undesired oscillations. These oscillations cannot be damped directly by the actuators and there are usually far more oscillation modes than independent control inputs. Therefore, the systems are inherently underactuated and the control of such systems is an active field of research.

Often, two-design degree of freedom control is applied for the trajectory control of flexible multibody systems. If the feedforward control is accurate, simple feedback control strategies are usually sufficient. An inverse model is an excellent choice for computing the feedforward control. However, many flexible multibody systems are non-minimum phase systems with unstable internal dynamics. Therefore, the systems must be analyzed with care in order to choose a suitable inversion method.

In this contribution, the inverse model is written in terms of servo-constraints [1]. This methods enables a simple incorporation of the desired trajectory similar to geometric constraints. The servo-constraints append the equations of motion such that

$$\dot{\boldsymbol{y}} = \boldsymbol{Z}(\boldsymbol{y})\boldsymbol{v} \tag{1}$$

$$\boldsymbol{M}(\boldsymbol{y},t)\dot{\boldsymbol{v}} + \boldsymbol{k}(\boldsymbol{y},\boldsymbol{v},t) = \boldsymbol{q}(\boldsymbol{y},\boldsymbol{v},t) + \boldsymbol{C}(\boldsymbol{y},t)^{\mathrm{T}}\boldsymbol{\lambda} + \boldsymbol{B}(\boldsymbol{y})\boldsymbol{u} \tag{2}$$

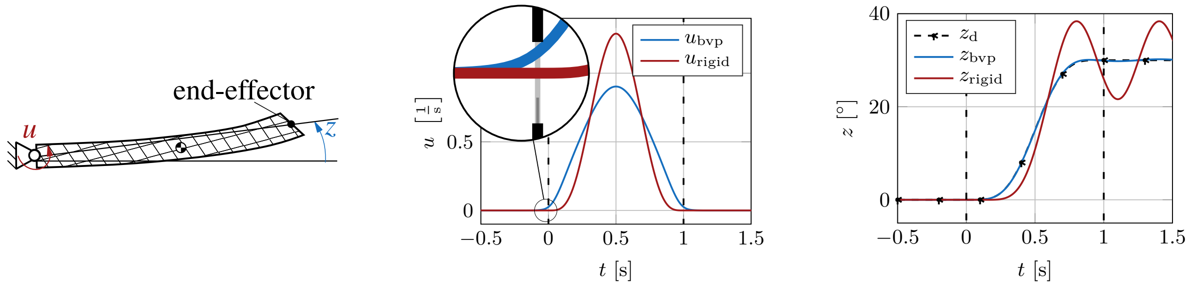
$$\boldsymbol{c}(\boldsymbol{y},t) = \boldsymbol{0} \tag{3}$$

$$\boldsymbol{s}(\boldsymbol{y},t) = \boldsymbol{z} - \boldsymbol{z}_{\mathrm{d}}(t) = \boldsymbol{0} \tag{4}$$

with $\boldsymbol{Z}$ describing the kinematic relationship between the positions $\boldsymbol{y}$ and velocities $\boldsymbol{v}$, the mass matrix $\boldsymbol{M}$, the Coriolis, centrifugal and gyroscopic forces $\boldsymbol{k}$, the applied forces $\boldsymbol{q}$ and the input distribution matrix $\boldsymbol{B}$. The Lagrange multipliers $\boldsymbol{\lambda}$ enforce the constraints $\boldsymbol{c}$ with the constraint gradient $\boldsymbol{C}$. The system output $\boldsymbol{z}$ is enforced to be equal to the desired output $\boldsymbol{z}_{\mathrm{d}}$ by the servo-constraint $\boldsymbol{s}$. While the first three equations represent the forward dynamics, the complete set of differential algebraic equations (DAEs) (1)-(4) represents the inverse model and can be solved for the system input $\boldsymbol{u}$ necessary to move the system on the desired trajectory $\boldsymbol{z}_{\mathrm{d}}$.

For minimum phase systems, the DAEs (1)-(4) can be solved forward in time. However, many flexible systems are non-minimum phase systems. In this case, stable system inversion must be applied. Then, a boundary value problem can be stated in terms of the inverse model DAEs in order to find a bounded solution to the inverse model problem [2]. While it is possible to solve the exact boundary value problem for small scale systems, this is usually not feasible for complex flexible multibody systems. Therefore, an approximation of the exact boundary value problem is proposed in this contribution. The simplification enables the application of stable system inversion to far more complex systems.

As an application example, flexible multibody systems are modeled using the absolute nodal coordinate formulation (ANCF) [4]. This enables the modeling of large deformations and large rigid body rotations. The equations are rather complex and a complete analytical analysis of the inverse model problem is not feasible. In a first example, a flexible manipulator with one link is considered, see Fig. 1 (a). The system input is a torque applied to the left joint, while the system output z is the angle of the end-effector. The desired trajectory is a smooth transition from the angle $z = 0°$ to $z = 30°$. The system inversion is performed using the approximated boundary value problem and the results are shown in Figs. 1 (b)-(c) with the index *bvp* [3]. The results are compared to the inversion of an equivalent rigid system, named *rigid*. It can be seen that the inversion of an equivalent rigid system is not sufficient for tracking, since it results in large oscillations around the desired trajectory.



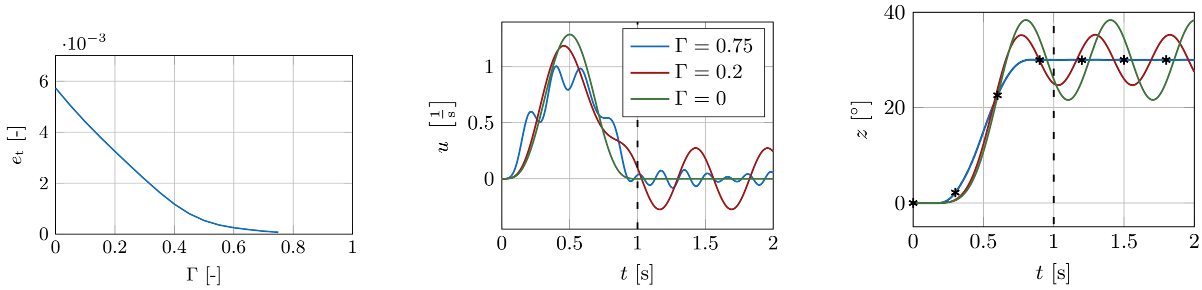
(a) Manipulator with one link. (b) System input u. (c) System output z.

Figure 1: Model and numerical results for system inversion for a flexible manipulator with one link.

While stable inversion is now possible for flexible multibody systems with the proposed approximation, it still involves solving a boundary value problem over the complete time domain of the trajectory. In practice, this is not achievable in real-time. Therefore, an output redefinition strategy is proposed for the flexible link manipulator. The new output $\tilde{z}$ is supposed to yield minimum phase behavior while approximating the original output. It is proposed to consider the new output

$$\tilde{z} = \Gamma z_{\text{flex}} + (1-\Gamma)\, z_{\text{rigid}} \tag{5}$$

with z_{rigid} describing an equivalent rigid or collocated output and z_{flex} describing the original output of the flexible system. Thereby, Γ is a parameter to choose a combination of both outputs, such that accurate tracking as well as minimum phase behavior is achieved. Due to the minimum phase behavior, the inverse model DAEs (1)-(4) can be integrated forward in time. This is potentially real-time capable for few number of ANCF elements. Numerical results are shown in Fig. 2. Thereby, the total tracking error is shown in Fig. 2(a) for different values Γ. The tracking error decreases for increasing Γ and therefore increasing the weight of the flexible output. The system is non-minimum phase for $\Gamma \gtrapprox 0.75$. The system input and output are shown in Figs. 2(b)-(c) for different values Γ. It can be seen that setting $\Gamma = 0.75$ yields almost perfect tracking. Residual tracking errors can be minimized by a adding a feedback controller.



(a) Total tracking error. (b) System input u. (c) System output z.

Figure 2: Numerical results for inversion of the flexible manipulator with one link with redefined system output $\tilde{z}$.

References

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