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Study of Gear Tooth Crack Analysis using Coupled Electromechanical System

Subrata Mukherjee[1], Vikash Kumar[2], Somnath Sarangi[3], Sanjeev Kumar[4] and Alok Kumar Verma[5]

[1,2,3,4] Mechanical Engineering Department
IIT Patna, Bihta, Bihar 801106, India
{subrata.re.me, vikashkumar7r, sanjeevkumar47,somsara}@gmail.com

[5] A*Star,
Connexis North Tower 138632, Singapore,
alok_kumar_verma@artc.a-star.edu.sg

Abstract

This paper presents an integrated method to study the gear tooth cracks based on electromechanical modelling of the spur gearbox. This study used improved time-varying mesh stiffness (IAM-TVMS) to develop the electromechanical model of the gearbox as an input parameter in the coupled electromechanical model, which is derived from the Lagrangian formulation and Rayleigh dissipative potential. Vibration responses of the electromechanical model are studied by an integrated approach of variable mode decomposition (VMD) and time synchronous averaging (TSA) where the four types of gear tooth conditions have been used, i.e., healthy tooth, tooth crack of 0.08 mm, tooth crack of 1.76 mm, and tooth crack of 2.64 mm. Based on the VMD-TSA responses, various gearbox conditions indicator such as time indicators, entropy indicators, etc., are used to predict the early gear tooth signatures. Results shows the significant improvements in the gear tooth crack analysis in very early age.

***Keywords**:* IAM-TVMS, electromechanical model, Lagrangian formulation, Rayleigh dissipative potential, gear tooth cracks, variable mode decomposition (VMD), time synchronous averaging (TSA), time indicators, entropy indicators.

1. Dynamic Modelling of Gearbox based on Improved Time Varying Mesh Stiffens (IAM-TVMS)

In this work, the IAM-TVMS model proposed in [1-4] has been taken for the calculation of the gear mesh stiffness of gear pair for both stages. This model is improved analytically by considering the following: (1) misalignment between root and base circle; (2) accurate transition curve with involute profile; (3) structural coupling effect when a nearby tooth is loaded; and (4) non-linear Hertzian contact between gear pairs. Fig. 1 shows the IAM-TVMS at different crack depths, which is further incorporated in the coupled electromechanical model of the gearbox presented in Fig. 2. Fig. 3 shows the vibration response obtained by solving coupled electromechanical equations of motion at different crack depths. Eq (1) is used to calculate the IAM-TVMS and its detailed explanation is given in [1,2].

$$K(t) = \begin{cases} \dfrac{1}{\left(\dfrac{1}{K_1^p} + \dfrac{1}{K_1^w} + \dfrac{1}{K_{h1}}\right)}, & \text{Single tooth engagement} \\[2ex] \dfrac{1}{\left(\dfrac{1}{K_1^p} + \dfrac{1}{K_1^w} + \dfrac{1}{K_{h1}}\right)} + \dfrac{1}{\left(\dfrac{1}{K_2^p} + \dfrac{1}{K_2^w} + \dfrac{1}{K_{h2}}\right)}, & \text{Double tooth engagement} \end{cases} \quad (1)$$

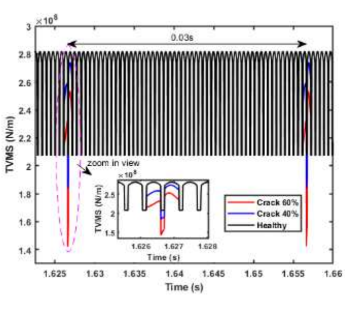

Fig. 1. TVMS response

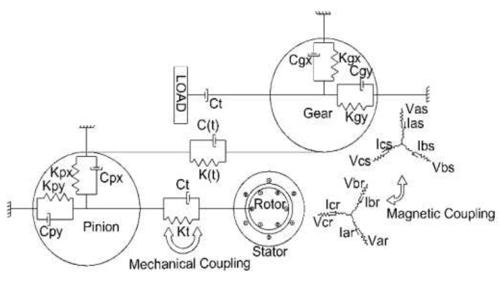

Fig. 2. Coupled electromechanical modelling of gearbox

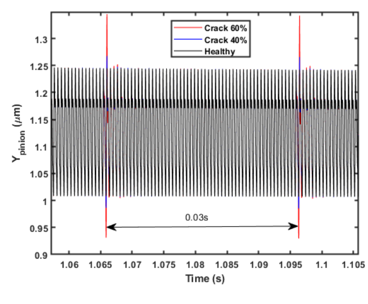

Fig. 3. Vibration response

2. VMD-TSA Method to study the gear tooth cracks

Variable mode decomposition (VMD) [5] is a type of signal processing technique mainly used to study the nonstationary vibration signal, which has the ability to filter out the signal non-stationarities. Time synchronous averaging (TSA) [6] is used to extract periodic waveforms from noisy data. This section demonstrates the represetation of the proposed concept to develop an integrated procedure to study dynamic resepose at a mono-component level for gear tooth crack analysis. Fig. 4 shows the VMD-TSA representaions of gear tooth cracks, where Figs. 4A and 4B show the VMD of a heatly tooth and a 2.64 mm crack tooth with the first three IMFs. Fig. 4a shows the TSA of the first IMF in the case of healthy teeth. 4b shows a cracked tooth of 0.08 mm, 4c shows a cracked tooth of 1.76 mm and 4d shows a cracked tooth of 2.64 mm. The increase in tooth crack is clearly visible from Figs 4a to 4d.This procedure will be used for the further study of gear tooth craks. Fig. 5 shows the Poincaré plot and bivariate histogram of the grid point distribution of healthy teeth and cracked teeth.

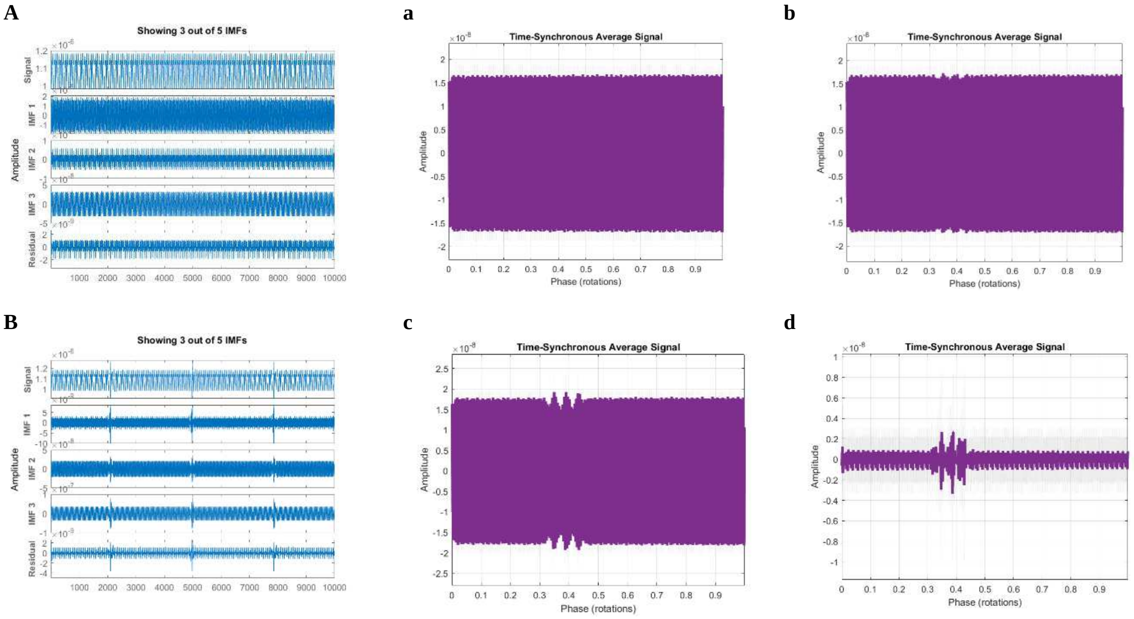

Fig. 4. (A) VMD-IMFs of healthy tooth (B) VMD-IMFs of tooth crack at 2.6 mm (a) VMF-TSA of healthy tooth (b) VMF-TSA of tooth crack 0.08 mm (c) VMF-TSA of tooth crack 1.76 mm and (d) VMF-TSA of tooth crack 2.64 mm

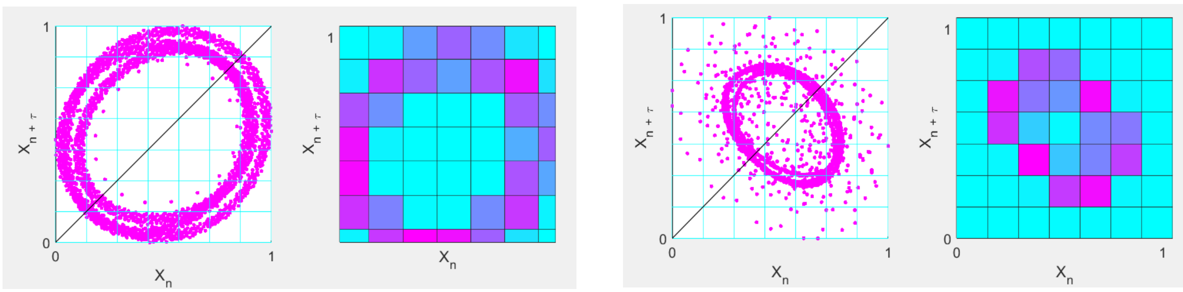

Fig. 5 Poincaré plot and bivariate histogram of the grid point distribution based on the healthy and tooth crack 2.64 mm at VMF1-TSA of 10000 datapoints

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Appendix A: Proposed Methodology and Complexity Measurements of Gear Tooth Cracks based on VMF1-TSA

Table 1 Calculation gear tooth complexity based on gridded distribution entropy (GDE) using VMF1-TSA

Healthy Tooth	Crack Level 1	Crack Level 2	Crack Level 3	Signal Conditions	Entropy
3.294487592	3.285693565	3.380573735	2.945351986	VMF1-TSA	GDE
3.3872513	3.364183727	3.285476362	3.00493343	VMF2-TSA	GDE
3.120159705	3.146708981	2.964425273	2.556762475	VMF3-TSA	GDE
2.895592893	2.918143336	2.94916544	2.594971594	VMF4-TSA	GDE
0.325677125	1.263760753	0.528615714	0.833356303	VMF5-TSA	GDE