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NOC based Dynamic Modelling of Multiphysics Systems: Studies on a DC Motor

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1 Introduction

Multiphysics systems involve components and structures that are governed by physical laws of different energy domains. For example, electrical circuits are analyzed using Kirchhoff's voltage and current laws whereas mechanical systems involve Newtons-Euler equations of motion. Often systems involving multiple physical laws are encountered like a "DC motor driving a hydraulic pump" or a "Thermal engine driving a muffler". Dynamic modelling of such systems has always been approached from the perspective of power and energy. One such method encountered in literature is Bond Graph technique that was introduced by H.M. Paynter [1]. It is a graphical tool that tries to capture the energy of whole system and portrays the system using power bonds which are manifestations of constraints in the system [2]. Governing ordinary differential equations and physical insights of the system can be algorithmically derived from the pictorial representation. It considers "efforts" and "flows" as generalized variables representing both the domains [1]. Schaft et al. [3] had shown that interconnected systems can also be modelled as port Hamiltonian systems with efforts and flows being orthogonal subspaces at the junctions of power exchange. Little has been studied on DAE formulation for such Multiphysics systems. Angeles and Lee [4], Saha and Angeles [5] had shown that when a constrained mechanical system is formulated as a DAE model, a matrix called Natural Orthogonal Complement can be derived whose column space spans the null space of the algebraic constraints. This matrix can be used to get a minimal order form which are essentially the unconstrained differential equations of motion for the system in hand. This paper presents an application of the NOC matrix to a DAE model of an electromechanical system.

2 Modelling

Fig.1. shows the equivalent circuit of a DC motor connected to a rigid mechanical load. It is well established in literature that forces and voltages, currents and velocities are analogous quantities from electrical and mechanical domain [2][3]. Hence a common flow space is considered for both domains and a flow vector is defined as $\boldsymbol{t} \equiv (I \quad \omega)^T$ where I and ω represent the motor armature current and its angular velocity respectively. The external effort vector is given by $\boldsymbol{E} \equiv (V \quad -\tau_L)^T$where V and τ_l signify applied voltage and load torque on the motor. Effort vector at the junction is represented as $\boldsymbol{E}' \equiv (-U_b \quad \tau_e)^T$, where U_b and τ_e denote the motor back emf and its generated electromagnetic torque.

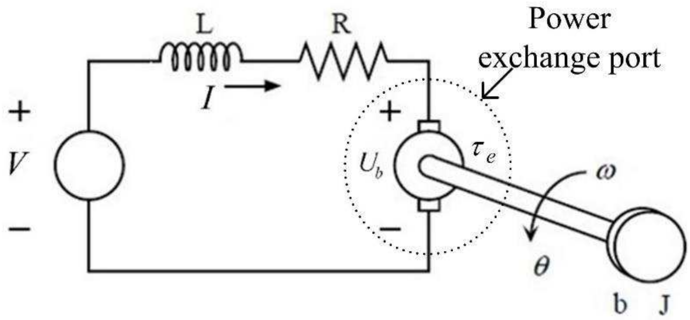

Figure 1: Equivalent circuit of a DC motor

A unified representation of the Kirchhoff's Voltage law and Newton-Euler equations of motion is adopted using a matrix-vector form as shown in equation (1). At the junction of power exchange effort vector can be obtained from the power conservation equation (2).

$$\begin{pmatrix} L & 0 \\ 0 & J \end{pmatrix}\begin{pmatrix} \dot{I} \\ \dot{\omega} \end{pmatrix} + \begin{pmatrix} R/L & 0 \\ 0 & b/J \end{pmatrix}\begin{pmatrix} L & 0 \\ 0 & J \end{pmatrix}\begin{pmatrix} I \\ \omega \end{pmatrix} = \begin{pmatrix} V \\ -\tau_L \end{pmatrix} + \begin{pmatrix} -U_b \\ \tau_e \end{pmatrix} \tag{1}$$

$$U_b I - \tau_e \omega = 0 \tag{2}$$

Here the, Natural Orthogonal Complement matrices $\boldsymbol{N_1}$and $\boldsymbol{N_2}$ are defined as relationships of flow vector with the flow variable of interest, thus $\boldsymbol{t} = \boldsymbol{N_1} I$ and $\boldsymbol{t} = \boldsymbol{N_2}\omega$ where , $\boldsymbol{N_1} = \begin{pmatrix} 1 & \frac{\tau_e}{U_b} \end{pmatrix}^T$ and $\boldsymbol{N_2} = \begin{pmatrix} 1 & \frac{U_b}{\tau_e} \end{pmatrix}^T$. A more general form of flow and effort based model for DC-motor is given by equation (3) and (4).

$$\boldsymbol{G\dot{t}} + \boldsymbol{WGt} = \boldsymbol{E} + \boldsymbol{E'} \tag{3}$$

$$\boldsymbol{t^T E'} = 0 \tag{4}$$

The diagonal matrix $\boldsymbol{G} \equiv \begin{pmatrix} L & 0 \\ 0 & J \end{pmatrix}$ is the systems "Storage matrix" which captures the energy storing elements involved in multiple domains like armature inductance (L) and equivalent mechanical inertia (J), whereas the diagonal matrix $\boldsymbol{W} \equiv \begin{pmatrix} R/L & 0 \\ 0 & b/J \end{pmatrix}$ represents the systems "Reaction time matrix" that reveals the systems time constants from electrical (R/L) and mechanical domain (b/J). Matrix $\boldsymbol{WG} \equiv \begin{pmatrix} R & 0 \\ 0 & b \end{pmatrix}$ denotes the "Dissipation matrix" that unifies energy dissipating elements from both fields like armature resistance (R) and viscous friction coefficient (b). Using $\boldsymbol{N_1}$and $\boldsymbol{N_2}$ the generalized flow model can be reduced to the dynamic equation corresponding to a specific flow variable as shown in equations (5) and (6).

$$\boldsymbol{N_1^T G\dot{t}} + \boldsymbol{N_1^T WGt} = \boldsymbol{N_1^T E} \Leftrightarrow L\dot{I} + RI + U_b = V \tag{5}$$

$$\boldsymbol{N_2^T G\dot{t}} + \boldsymbol{N_2^T WGt} = \boldsymbol{N_2^T E} \Leftrightarrow J\dot{\omega} + b\omega + \tau_L = \tau_e \tag{6}$$

3 Conclusion

In this paper an approach for formulating the constrained equation of motion for a multidomain system was shown using an example of DC motor connected to a rigid mechanical load, which is governed by laws of mechanical and electrical domain. The detailed study on DC motor connected to rigid and flexible links with results and numerical simulations will be provided in the full paper.

References

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