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DeNOC based recursive formulation to model ground impact of a biped

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1 Dynamic modelling

In this work, equations of motion of a biped are derived using the method of Decoupled Natural Orthogonal Complement (DeNOC) matrices [1]. Using the DeNOC method, biped is modelled as a 3 Degrees Of Freedom (DOF) serial chain system (Figure 1) using recursive relations, to express the kinematic constraints[2]. The Stance leg is assumed to be straight and has 1 DOF, whereas, the swing leg consists of 2 DOF's. The Euler-Lagrange methodology is used in [2] for dynamic modelling of the biped.

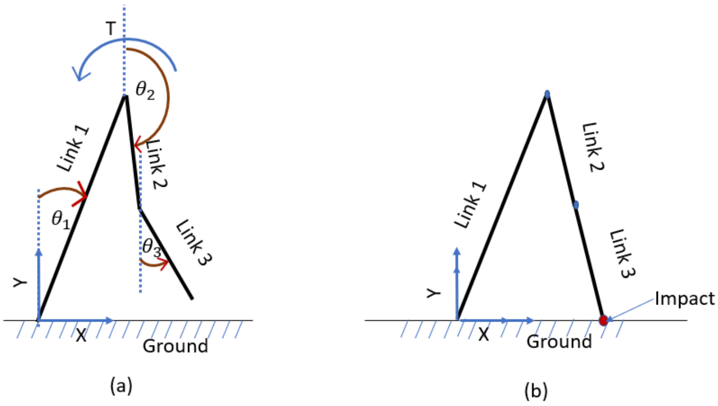

Figure 1: Kinematic structure of the biped system: (a) during swing phase (b) during ground impact.

For planar 3 DOF biped system, Newton-Euler equations of motion after pre-multiplication with **DeNOC** matrices can be written as,

$$N^T M\dot{\mathbf{t}} + N^T W M\mathbf{t} = N^T \mathbf{w} \tag{1}$$

In equation 1, $M = diag[M_1, M_2, M_3]$, $W = diag[W_1, W_2, W_3]$, $\mathbf{w} = [\mathbf{w}_1{}^T, \mathbf{w}_2{}^T, \mathbf{w}_3{}^T]^T$. N is decoupled as N=N_lN$_d$ [1]. $\mathbf{t} = [\mathbf{t}_1{}^T, \mathbf{t}_2{}^T, \mathbf{t}_3{}^T]^T$ is twist vector, which is related with $\dot{\boldsymbol{\theta}} = [\dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3]^T$ as $\mathbf{t} = N\dot{\boldsymbol{\theta}}$ [1] ,

$$M_i = \begin{pmatrix} I_i & \mathbf{0}_{3\times3} \\ \mathbf{0}_{3\times3} & m_i \mathbf{1}_{3\times3} \end{pmatrix} \quad W_i = \begin{pmatrix} \tilde{\boldsymbol{\omega}}_i & \mathbf{0}_{3\times3} \\ \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} \end{pmatrix} : i = 1,2,3. \tag{2}$$

Parameters having i as subscript refer to parameters corresponding to centre of gravity (CG) of i^{th} link. $\mathbf{t}_i = [\boldsymbol{\omega}_i{}^T, \mathbf{v}_i{}^T]^T$ and $\mathbf{w}_i = [\mathbf{n}_i{}^T, \mathbf{f}_i{}^T]^T$ are the twist and wrench vectors respectively, ω_i=angular velocity, $\mathbf{v}_i$= linear velocity, $\mathbf{n}_i$=moment, $\mathbf{f}_i$=force. M_i=mass matrix, I_i= mass moment of inertia m_i= mass. $\tilde{\boldsymbol{\omega}}_i$=skew-symmetric form of angular velocity vector $\boldsymbol{\omega}_i$.

2 Ground impact modelling

Ground impact modelling refers to calculation of change in joint velocities due to impact of the biped with ground. To calculate change in joint velocities, law of angular momentum conservation is utilized. In this section, a computationally efficient recursive method to calculate generalized momentum vector at all joints of a serial chain system is proposed. For general n-link serial chain system, summation of 6×1 generalized momentum vectors of link n, $n-1$,... i at i^{th} joint can be written as,

$$\sum_{k=i}^{n} \mathbf{P}_k = \begin{pmatrix} \mathbf{1}_{3\times3} & \sum_{j=i}^{n-1} \tilde{\mathbf{a}}_j \\ \mathbf{0}_{3\times3} & \mathbf{1}_{3\times3} \end{pmatrix} \begin{pmatrix} I_n \boldsymbol{\omega}_n + m_n(\boldsymbol{b}_n \times \boldsymbol{v}_n) \\ m_n \boldsymbol{v}_n \end{pmatrix} + \begin{pmatrix} \mathbf{1}_{3\times3} & \sum_{j=i}^{n-2} \tilde{\mathbf{a}}_j \\ \mathbf{0}_{3\times3} & \mathbf{1}_{3\times3} \end{pmatrix} \begin{pmatrix} I_{n-1} \boldsymbol{\omega}_{n-1} + m_{n-1}(\boldsymbol{b}_{n-1} \times \boldsymbol{v}_{n-1}) \\ m_{n-1} \boldsymbol{v}_{n-1} \end{pmatrix}$$
$$+ \ldots + \begin{pmatrix} I_i \boldsymbol{\omega}_i + m_i(\boldsymbol{b}_i \times \boldsymbol{v}_i) \\ m_i \boldsymbol{v}_i \end{pmatrix} \tag{3}$$

Where $\tilde{\mathbf{b}}_i$= skew-symmetric form of vector $\mathbf{b}_i$ (Vector from origin of i^{th} link to CG of i^{th} link), $\tilde{\mathbf{a}}_i$= skew-symmetric form of vector $\mathbf{a}_i$ (Vector from origin of i^{th} link to origin of $(i+1)^{th}$ link). Other parameters denote same as they are mentioned before.

From equation 3, 6×1 generalized momentum vector corresponding to i^{th} joint can be recursively written as,

$$\mathbf{P}_i = (1 - A_i)\left(\sum_{k=i+1}^{n} \mathbf{P}_k\right) + B_i M_i \mathbf{t}_i \tag{4}$$

Where B_i and A_i are,

$$B_i = \begin{pmatrix} \mathbf{1}_{3\times3} & \tilde{\mathbf{b}}_i \\ \mathbf{0}_{3\times3} & \mathbf{1}_{3\times3} \end{pmatrix} \qquad A_i = \begin{pmatrix} \mathbf{1}_{3\times3} & \tilde{\mathbf{a}}_i \\ \mathbf{0}_{3\times3} & \mathbf{1}_{3\times3} \end{pmatrix}$$

Suppose twist vector after impact for i^{th} link is $\mathbf{t}_i^{+}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$. Angular momentum along axis of rotation can be found by pre-multiplying equation 4 with $\mathbf{p}_i^T$ at instants of before and after impact, where $\mathbf{p}_i^T = [\boldsymbol{e_i}^T, \mathbf{0}_{3\times1}^T]$ ($\mathbf{e}_i$ is axis of rotation of i^{th} link). For planar 3 DOF serial chain system, $\boldsymbol{e_1} = \boldsymbol{e_2} = \boldsymbol{e_3} = [0,0,1]^T$. Since no change in external torques at joints during impact, angular momentum is conserved at all joints. For the biped system discussed above, Impact between foot and ground is assumed as a perfectly inelastic collision i.e relative velocity between foot and ground becomes 0 after impact. After applying law of conservation of angular momentum, joint velocities after impact along axis of rotation can be found out as,

$$\dot{\boldsymbol{\theta}}^{+} = \dot{\boldsymbol{\theta}}^{-} - (GIM)^{-1}\boldsymbol{h} \tag{5}$$

Where GIM is 3×3 general inertia matrix of the biped, obtained by both Euler-Lagrange and DeNOC method, $\dot{\boldsymbol{\theta}}$ and $\dot{\boldsymbol{\theta}^{+}}$ are 3×1 joint velocity vectors before and after ground impact respectively. $\boldsymbol{h}$, a 3×1 vector, is the difference in angular momentum between after and before ground impact along Z-axis.

3 Results

Results obtained using DeNOC methodology (equation 1) and the recursive method for momentum calculation (equation 5) are validated with results obtained by Euler-Lagrange method as per parameters used in [2]. From figure 2 it is observed that same closed phase portrait contour containing θ_1,θ_3 , $\dot{\theta}_1$ and

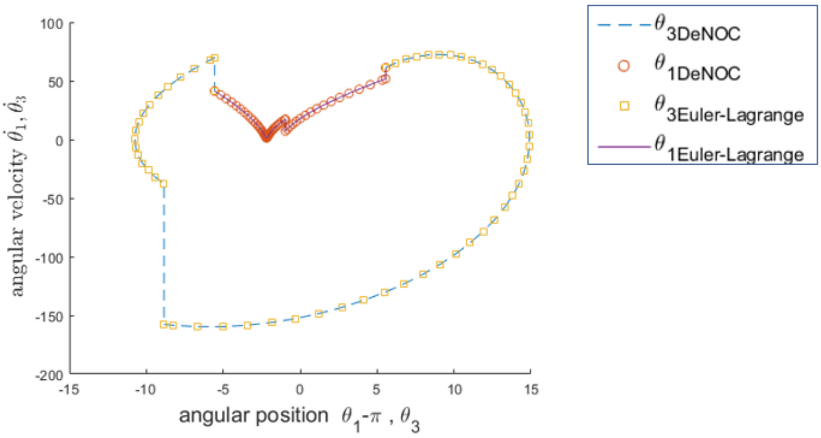

Figure 2: Phase portrait of biped system.

$\dot{\theta}_3$ are obtained from both the Euler-Lagrange and DeNOC methodology. Steep rise/fall in joint velocities indicate instances of ground impact [3]. So, the same results of [2] are obtained using a methodology which is computationally more efficient than Euler-Lagrange for dynamics and ground impact modelling of the biped. This method can be used instead of conventional Euler-Lagrange methodology for dynamics and ground impact modelling for systems having higher no of links, to reduce computational complexity.

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